Evolution of damage and plasticity in a variational framework without flow rule assumptions

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November 12th 2020

Dmitry Timofeev, Emilio Barchiesi, Anil Misra, Luca Placidi (in press). Hemivariational continuum approach for granular solids with damage-induced anisotropy evolution. MATHEMATICS AND MECHANICS OF SOLIDS, ISSN: 1081-2865, doi: 10.1177/1081286520968149

OUTLINE

- Introduction
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- Variational principle
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  - Governing equations
- Results
  - Homogeneous case
  - Non-homogeneous case
- Conclusions and outlook

INTRODUCTION

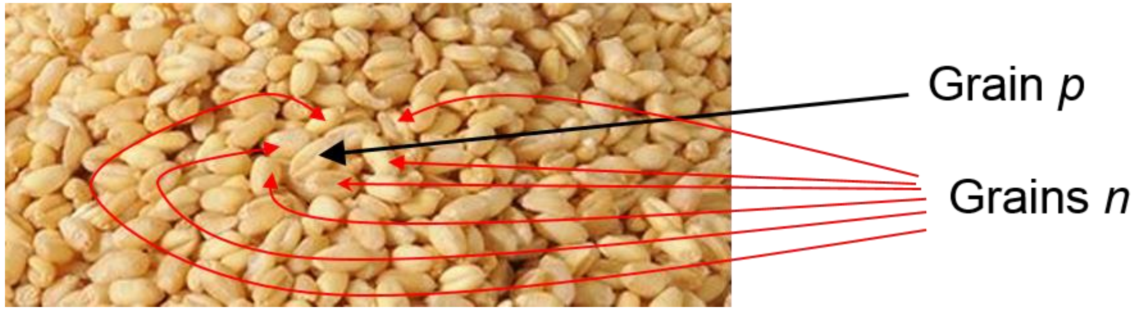
The class of granular materials spans a very wide spectrum of typologies

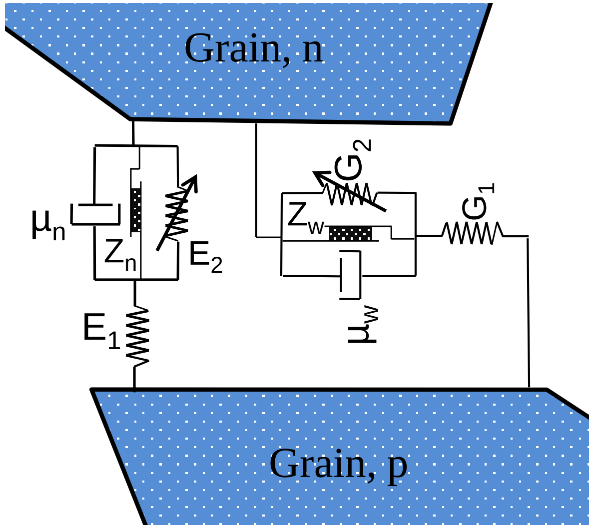
INTRODUCTION

The behavior of granular materials, at the macro-scale, ranges from

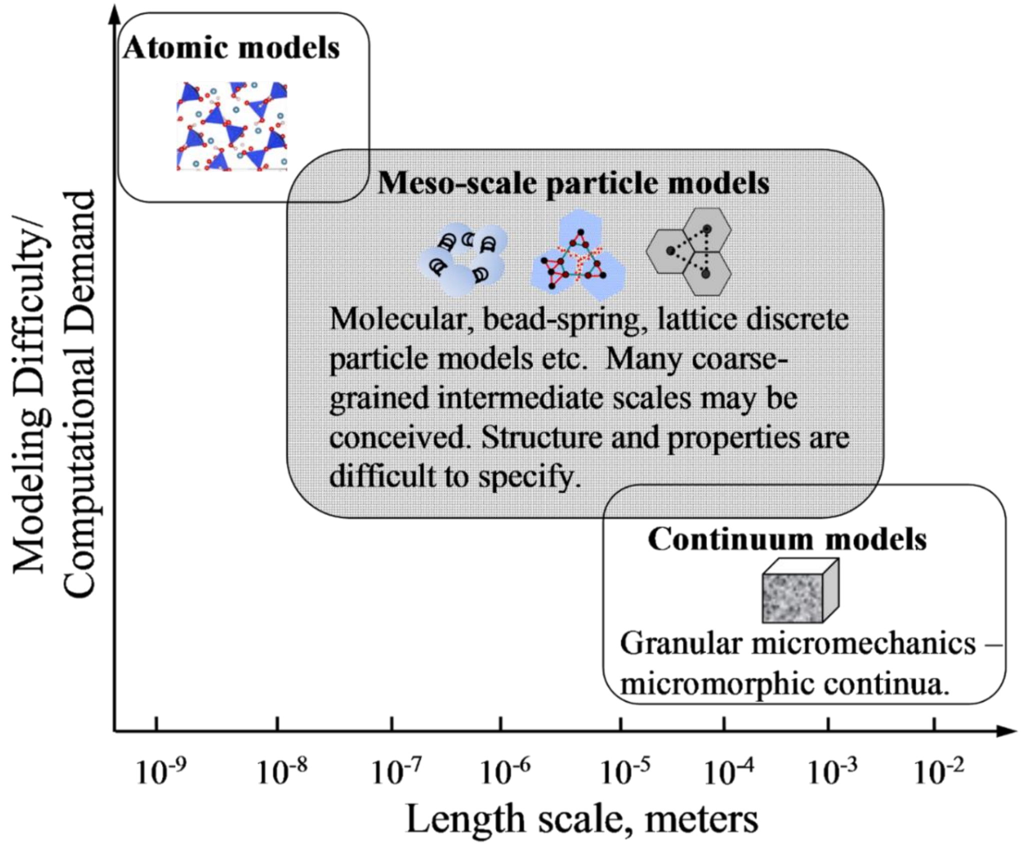
- strain softening to
- inherent and loading induced anisotropy,
- localized strains, fractures, etc.

Moreover, it is strongly affected by interactions at the grain-scale





INTRODUCTION



Computational demand depends upon length scale according to this schematic picture. We therefore argue that for many problems in engineering and science, continuum description of mechanical behavior is sometimes desirable.

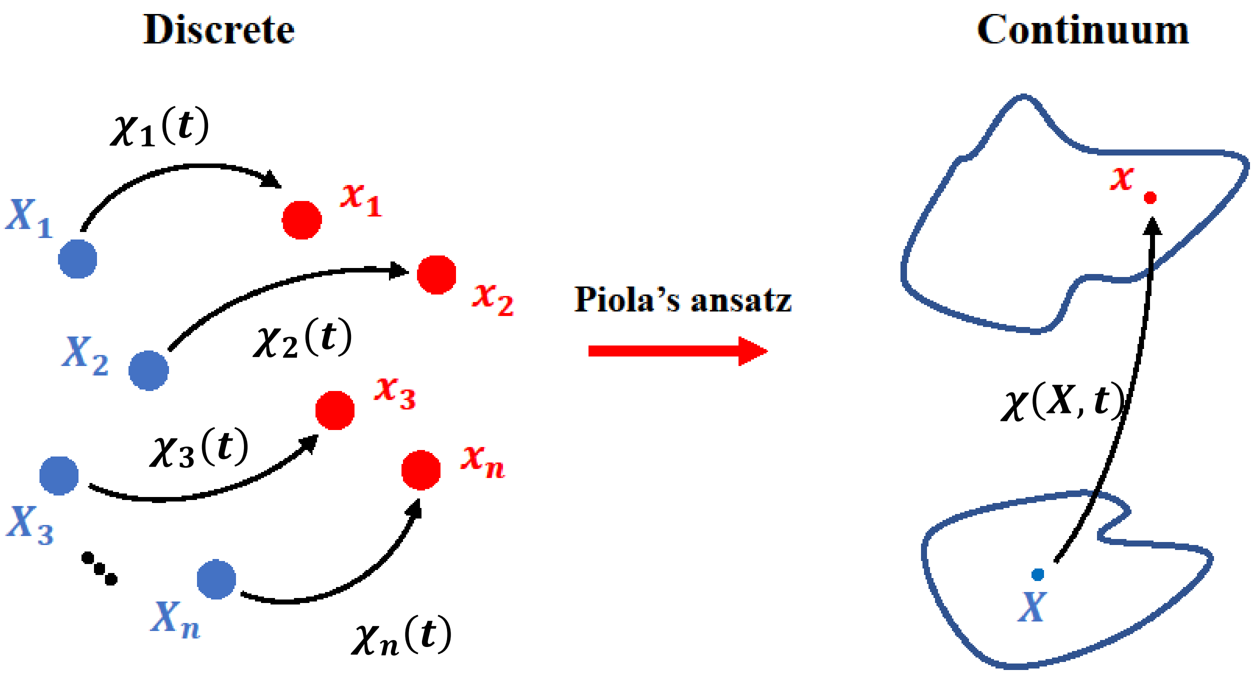
CHALLANGES

The challenge is the development of continuum models that
1) represent micro-scale effects of grain interactions,
2) describe mechanical properties at scales amenable to continuum description,
3) have far smaller computational needs than that of other grain-based approaches, such as Discrete Element Method DEM or Molecular Dynamics MD (which have their own issues)

OBJECTIVES

- Study mechanical behavior of materials with granular microstructures in presence of damage;
- Investigate effect of damage induced anisotropy;
- Overcome mesh dependency in numerical damage mechanics via second gradient;
- Develop, utilizing variational approach, continuum models which take into account unique dissipative features of the grain-scale mechano-morphology.

KINEMATICS AND PIOLA'S ANSATZ



Discrete model

Set of n grains:

$X_1, X_2, \dots, X_n \in \left(E^2\right)^n$

Placement and displacement:

$i = \overline{1,n}$,

$x_i = \chi_i(t), \qquad u_i(t) = x_i - X_i,$

Identification

The following Piola's ansatz is assumed:

$$u(X,t)_{|X=X_i} = u_i(t), \qquad i = \overline{1,n}$$

Continuous model

Continuous body B:

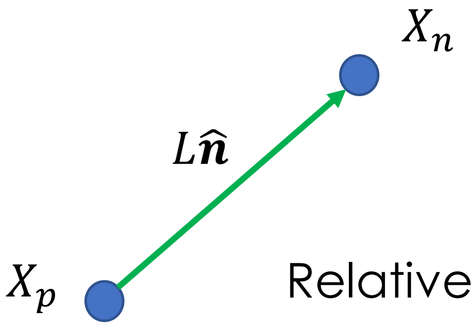
$X \in B \subset E^2$

Placement and displacement:

$\forall X \in B,$

$\chi(X,t), \qquad u(X,t) = \chi(X,t) - X,$

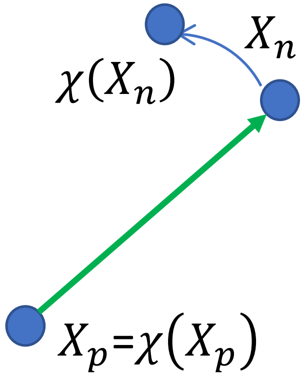
OBJECTIVE DEFORMATION MEASURES



$$X_n - X_p = L\hat{\boldsymbol{n}} \overset{\chi}{\rightarrow} \chi(X_n, t) - \chi(X_p, t),$$

Relative displacement: $\delta^{np}(t) = \big(\chi(X_n, t) - \chi(X_p, t)\big) - (X_n - X_p)$

is not objective. For example a rigid body rotation gives non zero relative displacement and zero strain energy



An example of objective relative displacement is the following:

$$\boldsymbol{u}^{np} = F^T\big(\chi(X_n, t) - \chi(X_p, t)\big) - (X_n - X_p), \qquad F = \nabla\chi$$

OBJECTIVE DEFORMATION MEASURES

We assume small value of the grain-pair distance *L* with respect to the characteristic size of the agglomaerate and therefore the validity of the following Taylor's expansion:

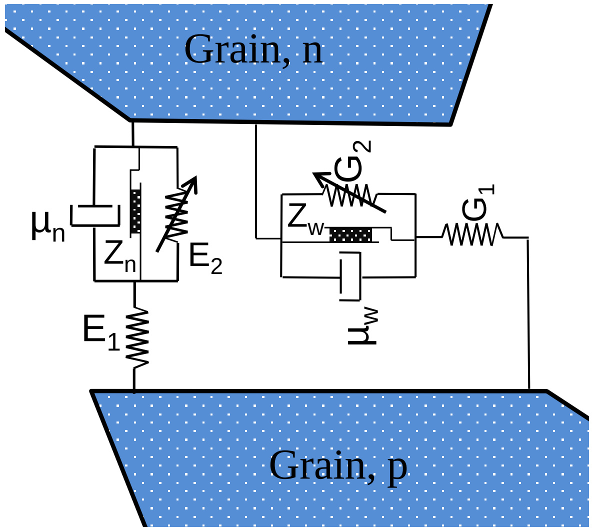
$$\chi(X_n,t) = \chi(X_p,t) + (\nabla\chi)_{|X_p} + \frac{1}{2}\left[\left(\nabla^2\chi\right)_{|X_p}(X_n - X_p)\right]\cdot(X_n - X_p)$$

By insertion of this expansion into the objective relative displacement we have, in index notation

$$u_i^{np} = 2LG_{ij}\,\hat{n}_j + \frac{1}{2}L^2 G_{ij,h}\hat{n}_j\hat{n}_h$$

where the definition of the Green-Saint-Venant strain tensor has been used

$$G = \frac{1}{2}\left(F^T F - I\right)$$



The normal relative displacement:

$$u_\eta = \boldsymbol{u}^{np}\cdot\hat{\boldsymbol{n}}$$

The tangential relative displacement:

$$\boldsymbol{u}_\tau = \boldsymbol{u}^{np} - (\boldsymbol{u}^{np}\cdot\hat{\boldsymbol{n}})\hat{\boldsymbol{n}}$$

OBJECTIVE DEFORMATION MEASURES

Squared normal relative displacement:

$$u_\eta = \boldsymbol{u}^{np} \cdot \hat{\boldsymbol{n}}$$

$$u_\eta = 2LG_{ij}\,\hat{n}_i\hat{n}_j + \frac{1}{2}G_{ij,h}\hat{n}_i\hat{n}_j\hat{n}_h$$

$$u_\eta^2 = 4L^2\hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b G_{ij}\,G_{ab} + 2L^3\hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b\hat{n}_c G_{ij}\,G_{ab,c} + \frac{1}{4}L^4\hat{n}_i\hat{n}_j\hat{n}_h\hat{n}_a\hat{n}_b\hat{n}_c G_{ij,h}G_{ab,c}$$

Squared tangential relative displacement:

$$\boldsymbol{u}_\tau = \boldsymbol{u}^{np} - (\boldsymbol{u}^{np} \cdot \hat{\boldsymbol{n}})\hat{\boldsymbol{n}}$$

$$|\boldsymbol{u}_\tau|^2 = u_\tau^2 = \boldsymbol{u}^{np} \cdot \boldsymbol{u}^{np} - (\boldsymbol{u}^{np} \cdot \hat{\boldsymbol{n}})^2$$

$$u_\tau^2 = 4L^2G_{ij}\,G_{ab}\left(\delta_{ia}\hat{n}_j\hat{n}_b - \hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b\right) + 2L^3G_{ij}\,G_{ab,c}\left(\delta_{ia}\hat{n}_j\hat{n}_b\hat{n}_c - \hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b\hat{n}_c\right) + \frac{1}{4}L^4G_{ij,h}G_{am,n}\left(\delta_{ia}\hat{n}_j\hat{n}_h\hat{n}_m\hat{n}_n - \hat{n}_i\hat{n}_j\hat{n}_h\hat{n}_a\hat{n}_b\hat{n}_c\right)$$

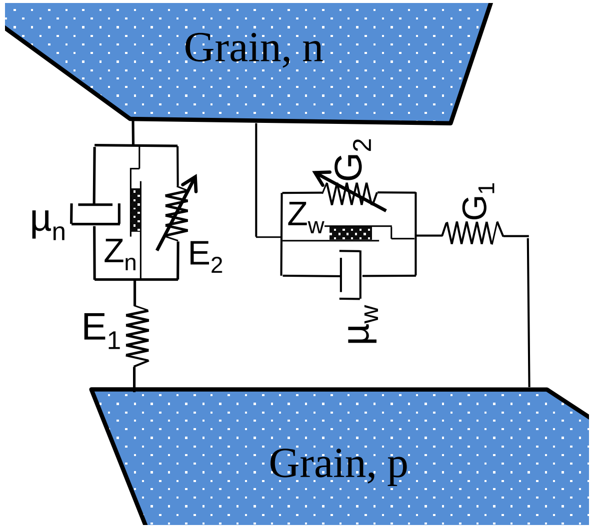
The elastic energy function for a given couple of grains:

$$U = \frac{1}{2}k_{\eta D}u_\eta^2 + \frac{1}{2}k_{\tau D}u_\tau^2$$

DAMAGES, STIFFNESSES AND STRAIN ENERGY

k_η^t - normal stiffness in tension
k_η^c - normal stiffness in compression

are reduced with $D_\eta \in [0,1]$ - normal damage



Normal stiffness in compression is higher than in tension ($k_\eta^c \gg k_\eta^t$)

k_τ - tangential stiffness is reduced with $D_\tau \in [0,1]$ - tangential damage

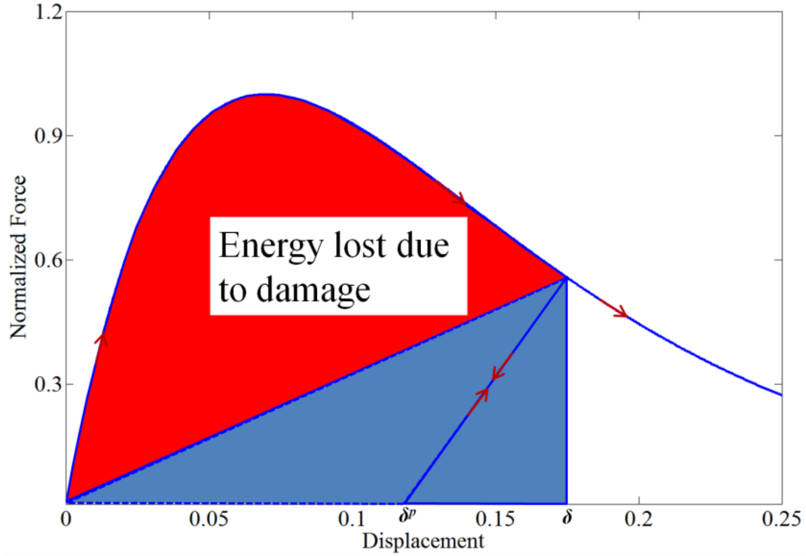
Thus, the damaged stiffnesses are

$$k_{\eta D} = k_\eta\,(1 - D_\eta) = k_\eta^t(1 - D_\eta)H(u_\eta) + k_\eta^c(1 - D_\eta)H(-u_\eta), \qquad k_{\tau D} = k_\tau(1 - D_\tau)$$

where the Heaviside function H has been used.

The elastic energy function for a given couple of grains:

$$U = \frac{1}{2}k_{\eta D}u_\eta^2 + \frac{1}{2}k_{\tau D}u_\tau^2$$



Kinematics and constitutive assumptions

ELASTIC ENERGY FUNCTION

Total elastic energy is the sum over all the *N* grain pairs :

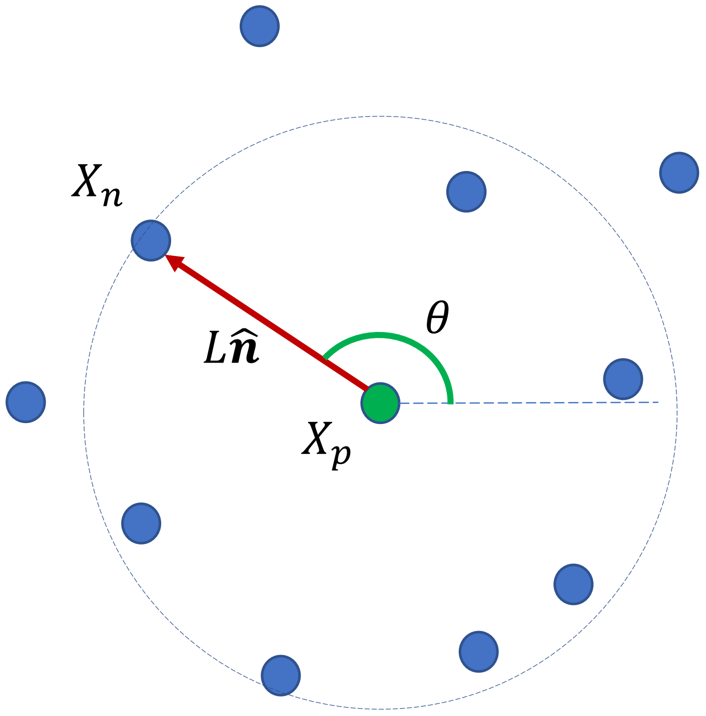
$$U^{tot} = \sum_{i=1}^{N}\left[\frac{1}{2}k_{\eta D,i}u_{\eta,i}^2 + \frac{1}{2}k_{\tau D,i}u_{\tau,i}^2\right]$$

Continuization is done by use the following homogenization rule

$$\sum_{i=1}^{N} a_i \rightarrow \int_{S^1} a$$

where S^1 is the unit circle. Thus, continuum total elastic energy is

$$U = \int_{S^1} \frac{1}{2}k_{\eta D}u_\eta^2 + \frac{1}{2}k_{\tau D}u_\tau^2$$



$$U = \int_{S^1} \frac{1}{2}k_\eta(1-D_\eta)\left(4L^2\hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b G_{ij}G_{ab} + 2L^3\hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b\hat{n}_c G_{ij}G_{ab,c}\right)$$
$$+ \int_{S^1} \frac{1}{2}k_\eta(1-D_\eta)\left(\frac{1}{4}L^4\hat{n}_i\hat{n}_j\hat{n}_h\hat{n}_a\hat{n}_b\hat{n}_c G_{ij,h}G_{ab,c}\right)$$
$$+ \int_{S^1} \frac{1}{2}k_\tau(1-D_\tau)\left(4L^2 G_{ij}G_{ab}\left(\delta_{ia}\hat{n}_j\hat{n}_b - \hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b\right) + 2L^3 G_{ij}G_{ab,c}\left(\delta_{ia}\hat{n}_j\hat{n}_b\hat{n}_c - \hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b\hat{n}_c\right)\right)$$
$$+ \int_{S^1} \frac{1}{2}k_\tau(1-D_\tau)\left(\frac{1}{4}L^4 G_{ij,h}G_{ab,c}\left(\delta_{ia}\hat{n}_j\hat{n}_h\hat{n}_b\hat{n}_c - \hat{n}_i\hat{n}_j\hat{n}_h\hat{n}_a\hat{n}_b\hat{n}_c\right)\right)$$

ELASTIC ENERGY FUNCTION

$$U = \frac{1}{2}\mathbb{C}_{ijab}G_{ij}G_{ab} + \mathbb{M}_{ijabc}G_{ij}G_{ab,c} + \frac{1}{2}\mathbb{D}_{ijhabc}G_{ij,h}G_{ab,c}$$

$\mathbb{C}$, $\mathbb{M}$ and $\mathbb{D}$ are the elasticity tensors of 4th , 5th and 6th rank respectively:

$$\mathbb{C}_{ijab} = 4L^2 \int_{S^1} k_\eta\left(1 - D_\eta\right)\hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b$$
$$+4L^2 \int_{S^1} k_\tau(1 - D_\tau)\left(\frac{1}{4}\left(\delta_{ia}\hat{n}_j\hat{n}_b + \delta_{ib}\hat{n}_j\hat{n}_a + \delta_{ja}\hat{n}_i\hat{n}_b + \delta_{jb}\hat{n}_i\hat{n}_a\right) - \hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b\right)$$

$$\mathbb{M}_{ijabc} = L^3 \int_{S^1} k_\eta\left(1 - D_\eta\right)\hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b\hat{n}_c$$
$$+L^3 \int_{S^1} k_\tau(1 - D_\tau)\left(\frac{1}{4}\left(\delta_{ia}\hat{n}_j\hat{n}_b + \delta_{ib}\hat{n}_j\hat{n}_a + \delta_{ja}\hat{n}_i\hat{n}_b + \delta_{jb}\hat{n}_i\hat{n}_a\right)\hat{n}_c - \hat{n}_i\hat{n}_j\hat{n}_a\hat{n}_b\hat{n}_c\right)$$

$$\mathbb{D}_{ijhabc} = \frac{1}{4}L^4 \int_{S^1} k_\eta\left(1 - D_\eta\right)\hat{n}_i\hat{n}_j\hat{n}_h\hat{n}_a\hat{n}_b\hat{n}_c$$
$$+\frac{1}{4}L^4 \int_{S^1} k_\tau(1 - D_\tau)\left(\frac{1}{4}\left(\delta_{ia}\hat{n}_j\hat{n}_b + \delta_{ib}\hat{n}_j\hat{n}_a + \delta_{ja}\hat{n}_i\hat{n}_b + \delta_{jb}\hat{n}_i\hat{n}_a\right)\hat{n}_h\hat{n}_c - \hat{n}_i\hat{n}_j\hat{n}_h\hat{n}_a\hat{n}_b\hat{n}_c\right)$$

DISSIPATION ENERGY

Dissipation energy is assumed to be additively decomposed:

$$W = W^{\eta} + W^{\tau}$$

The normal contribution:

$$W^{\eta} = \frac{1}{2} k_{\eta}^{c} \left(B_{\eta}^{c}\right)^{2} H(-u_{\eta}) \left[-D_{\eta} + \frac{2}{\pi} \tan\left(\frac{\pi}{2} D_{\eta}\right) \right] + k_{\eta}^{t} \left(B_{\eta}^{t}\right)^{2} H(u_{\eta}) \left[2 + (D_{\eta} - 1)(2 - \log(1 - D_{\eta}) + \left(\log(1 - D_{\eta})\right)^{2} \right]$$

The tangent contribution:

$$W^{\tau} = \frac{1}{2} k_{\tau} \left[B_{\tau}(u_{\eta})\right]^{2} [2 + (D_{\tau} - 1)(2 - \log(1 - D_{\tau}) + (\log(1 - D_{\tau}))^{2}]$$

Here $\left(B_{\eta}^{c}, B_{\eta}^{t}\right) \in \mathbb{R}_{+}^{2}$ are the characteristic lengths in compression and in tension, respectively.
B_{τ} is the characteristic length for tangent damage dissipation and it is assumed to depend on u_{η}:

$$B_{\tau} = B_{\tau}(u_{\eta}) = \begin{cases} B_{\tau 0}, & u_{\eta} \geq 0, \\ B_{\tau 0} - \alpha_{2} u_{\eta}, & \frac{1 - \alpha_{1}}{\alpha_{2}} B_{\tau 0} \leq u_{\eta} < 0, \\ \alpha_{1} B_{\tau 0}, & u_{\eta} < \frac{1 - \alpha_{1}}{\alpha_{2}} B_{\tau 0} \end{cases}$$

where α_1 and α_2 are further constitutive parameters.

ACTION FUNCTIONAL

The external world can exert forces expending power both on the scalar normal and on the vector tangent objective relative displacements, so that the external energy functional is

$$U^{ext} = F_{\eta}^{ext}\, u_{\eta} + \boldsymbol{F}_{\tau}^{ext} \cdot \boldsymbol{u}_{\tau},$$

where F_{η}^{ext} and $\boldsymbol{F}_{\tau}^{ext}$ are, respectively, the external scalar normal and vector tangent external forces.

The action functional is defined at the grain-level

$$\mathcal{E} = \int_{T_0}^{T_N} U + W - U^{ext}\,,$$

where T_0, T_N are instants of time.

$$T_t \in \{T_t\}_{t=0,\dots,N}, \qquad T_t \in \mathbb{R}, N \in \mathbb{N}$$
$$T_0 \leq T_1 \leq \dots \leq T_N$$

Variational principle

VARIATIONAL INEQUALITY

Fundamental kinematical quantities:

$$u_\eta, \boldsymbol{u}_\tau, D_\eta, D_\tau$$

Action functional

$$\mathcal{E} = \mathcal{E}(u_\eta, \boldsymbol{u}_\tau, D_\eta, D_\tau)$$

Variations of kinematical variables:

$$v = (\delta u_\eta, \delta \boldsymbol{u}_\tau) \in AV_t, \qquad \beta = (\delta D_\eta, \delta D_\tau) \in \mathbb{R}^{+2},$$

where AV_t is a set of admissible variations which is a subset of kinematically admissible displacements.

The increment at T_t

$$(\Delta u_\eta, \Delta \boldsymbol{u}_\tau, \Delta D_\eta, \Delta D_\tau)_{\mathrm{t}} = (u_\eta, \boldsymbol{u}_\tau, D_\eta, D_\tau)_t - (u_\eta, \boldsymbol{u}_\tau, D_\eta, D_\tau)_{t-1}$$

Variation of the action functional:

$$\delta\mathcal{E} = \mathcal{E}(u_\eta + \delta u_\eta, \boldsymbol{u}_\tau + \delta \boldsymbol{u}_\tau, D_\eta + \delta D_\eta, D_\tau + \delta D_\tau) - \mathcal{E}(u_\eta, \boldsymbol{u}_\tau, D_\eta, D_\tau)$$

$$\delta\mathcal{E} = \langle \mathcal{E}'(u_\eta, \boldsymbol{u}_\tau, D_\eta, D_\tau), (\delta u_\eta, \delta \boldsymbol{u}_\tau, \delta D_\eta, \delta D_\tau) \rangle$$

The variational principle is formulated as follows:

$$\langle \mathcal{E}'(u_\eta, \boldsymbol{u}_\tau, D_\eta, D_\tau), (\Delta u_\eta, \Delta \boldsymbol{u}_\tau, \Delta D_\eta, \Delta D_\tau) \rangle \le \langle \mathcal{E}'(u_\eta, \boldsymbol{u}_\tau, D_\eta, D_\tau), (v, \beta) \rangle$$

$$\forall v = (\delta u_\eta, \delta \boldsymbol{u}_\tau) \in AV_t, \qquad \forall \beta = (\delta D_\eta, \delta D_\tau) \in \mathbb{R}^{+2}$$

GOVERNING EQUATIONS

Euler-Lagrange equations:

$$-k_\eta(1-D_\eta)u_\eta - k_\tau B_\tau \frac{\partial B_\tau}{\partial u_\eta}\int_0^{D_\tau}[\log(1-x)]^2 dx + F_\eta^{ext} = 0$$

$$-k_\tau(1-D_\tau)\boldsymbol{u}_\tau + \boldsymbol{F}_\tau^{ext} = 0$$

Karush-Kuhn-Tucker (KKT) conditions:

$$\left[u_\eta^2 - \left(B_\eta^t\right)^2 H(u_\eta)[\log(1-D_\eta)]^2 - \left(B_\eta^c\right)^2 H(-u_\eta)\left[\tan\left(\frac{\pi}{2}D_\eta\right)\right]^2\right]\Delta D_\eta = 0$$

$$[u_\tau^2 - (B_\tau)^2[\log(1-D_\tau)]^2]\Delta D_\tau = 0$$

For both conditions it is either $[\ldots] = 0$ or $\Delta D_{\eta \text{ or } \tau} = 0$.

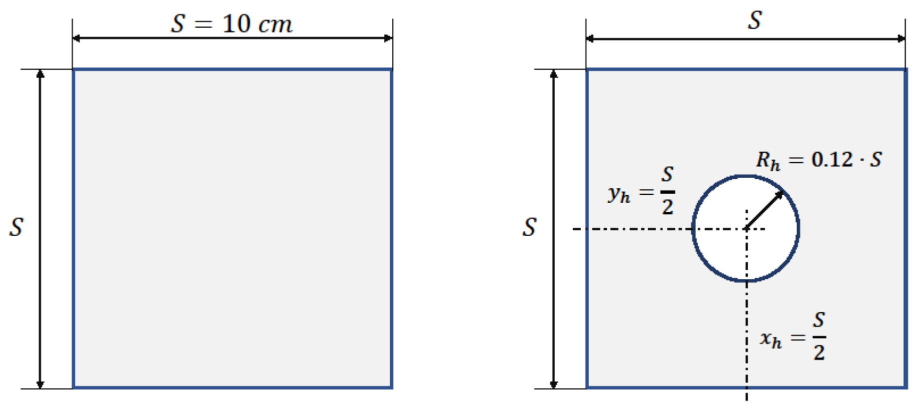
KKT conditions give us analytical expressions for damage

$$u_\eta > 0 \rightarrow D_\eta = 1 - \exp\left(-\frac{u_\eta}{B_\eta^t}\right)$$

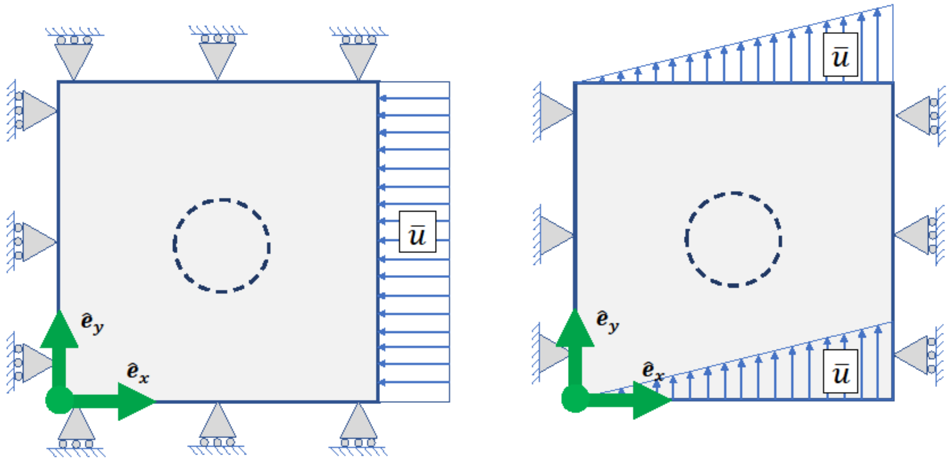
$$u_\eta < 0 \rightarrow D_\eta = \frac{2}{\pi}\arctan\left(-\frac{u_\eta}{B_\eta^c}\right)$$

$$D_\tau = 1 - \exp\left(-\frac{|\boldsymbol{u}_\tau|}{B_\tau}\right)$$

NUMERICAL EXPERIMENTS



Schematics of analyzed domains



Schematics of considered boundary conditions

Two square specimens in 2D, with side $S = 10$ [cm] are subjected to extension, compression and shearing tests, the quantity $\bar{u}$ increasing from 0 to $\bar{u}_{max}$

$L[m]$	$k_\eta^c \left[\frac{J}{m^4}\right]$	$k_\eta^t \left[\frac{J}{m^4}\right]$	$k_\tau \left[\frac{J}{m^4}\right]$	$B_\eta^c[m]$	$B_\eta^t[m]$	$B_{\tau 0}[m]$	α_1	α_2
0.01	3.5e14	3.5e13	3e13	3e-7	7e-8	5e-8	10	7

RESULTS: HOMOGENEOUS CASE

Extension-compression test:

$$u_1(X_1,X_2) = -\frac{\bar{u}}{S}X_1, \qquad u_2(X_1,X_2) = 0, \qquad \forall (X_1,X_2) \in [0,S]\times[0,S]$$

$$G = \begin{pmatrix} \frac{\bar{u}}{S}\left(-1+\frac{\bar{u}}{2S}\right) & 0 \\ 0 & 0 \end{pmatrix} \quad \rightarrow \quad \begin{aligned} u_\eta &= 2\bar{u}L\left(-1+\frac{\bar{u}}{2S}\right)\cos^2\theta \\ u_\tau^2 &= \left[2\bar{u}L\left(-1+\frac{\bar{u}}{2S}\right)\cos\theta\sin\theta\right]^2 \end{aligned}$$

Shearing test:

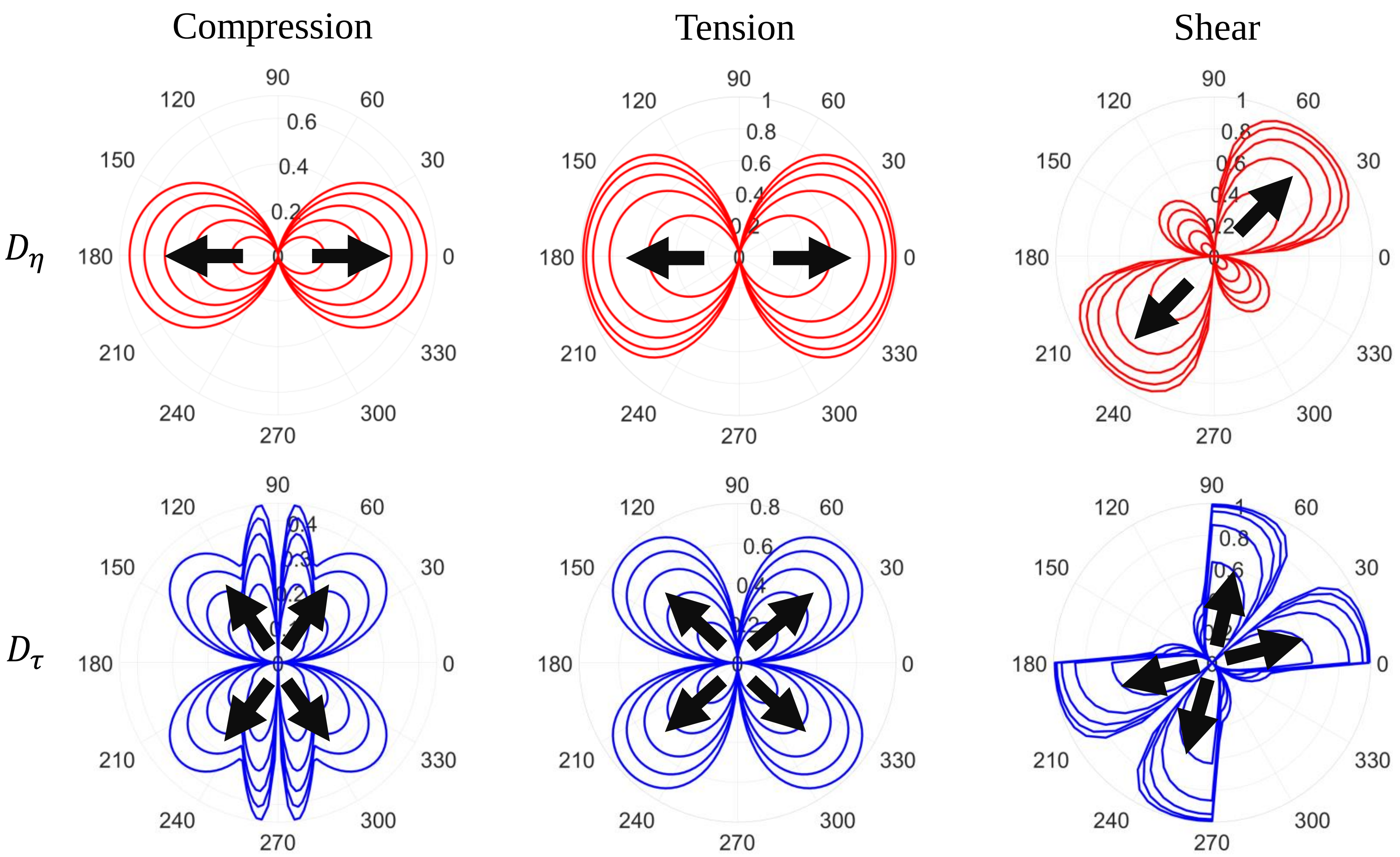
$$u_1(X_1,X_2) = 0, \qquad u_2(X_1,X_2) = \frac{\bar{u}}{S}X_1, \qquad \forall (X_1,X_2) \in [0,S]\times[0,S]$$

$$G = \begin{pmatrix} 0 & \frac{\bar{u}}{2S} \\ \frac{\bar{u}}{2S} & 0 \end{pmatrix} \quad \rightarrow \quad \begin{aligned} u_\eta &= 2\bar{u}\cos\theta\sin\theta \\ u_\tau^2 &= \bar{u}^2\left(1-4\cos^2\theta\sin^2\theta\right) \end{aligned}$$

This allows to calculate damage variables:

$$D_\eta = \begin{cases} 1-\exp\left(-\frac{u_\eta(\theta)}{B_\eta^t}\right), u_\eta(\theta) > 0 \\ \frac{2}{\pi}\arctan\left(-\frac{u_\eta(\theta)}{B_\eta^c}\right), u_\eta(\theta) < 0 \end{cases}, \qquad D_\tau = 1-\exp\left(-\frac{|u_\tau(\theta)|}{B_\tau}\right)$$

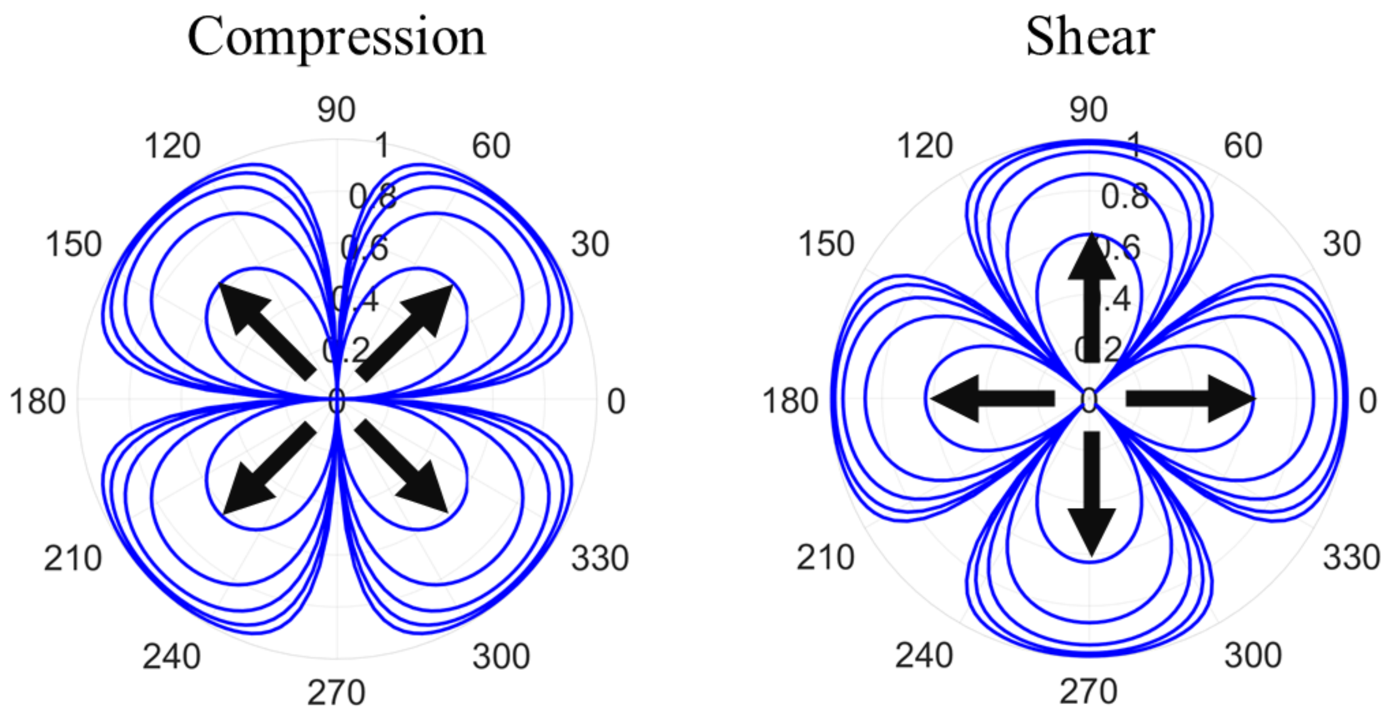
RESULTS: HOMOGENEOUS CASE



Polar plots of the damage variables D_η and D_τ for different homogeneous test cases and for increasing $\bar{u}$. Black arrows indicate directions of increasing $\bar{u}$.

Results

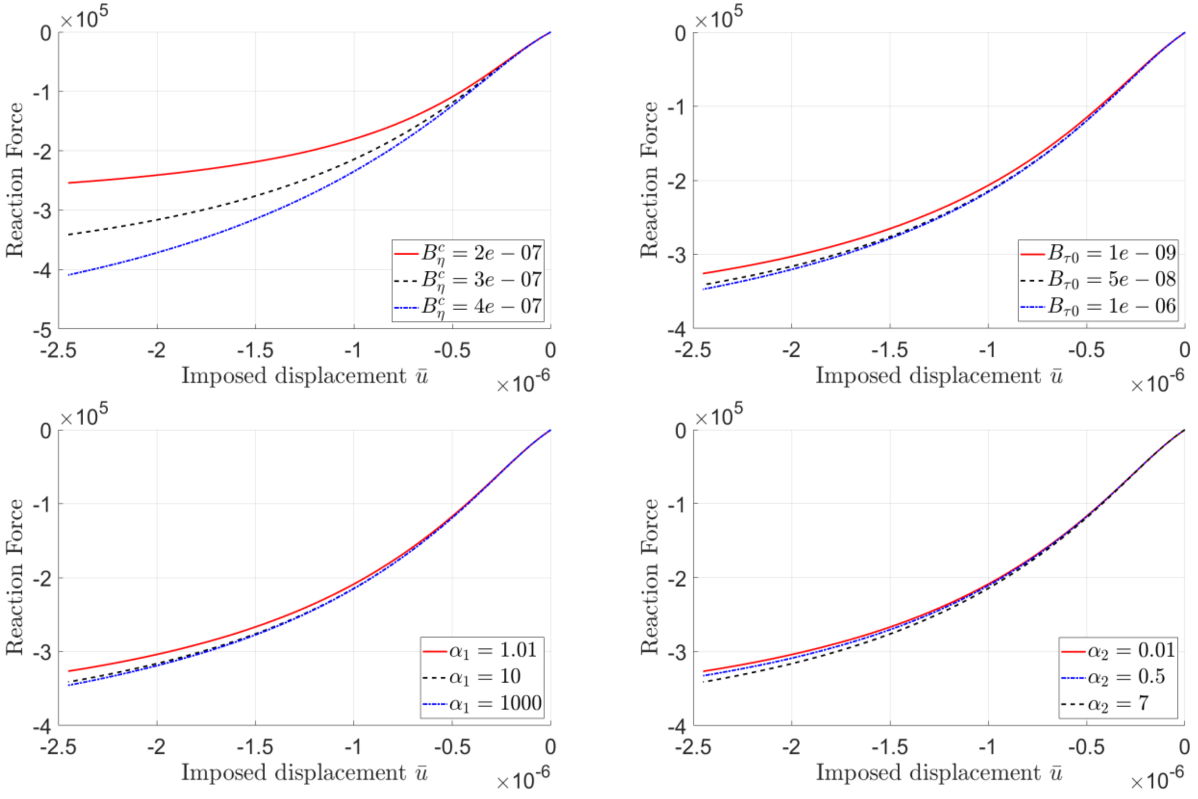
RESULTS: HOMOGENEOUS CASE



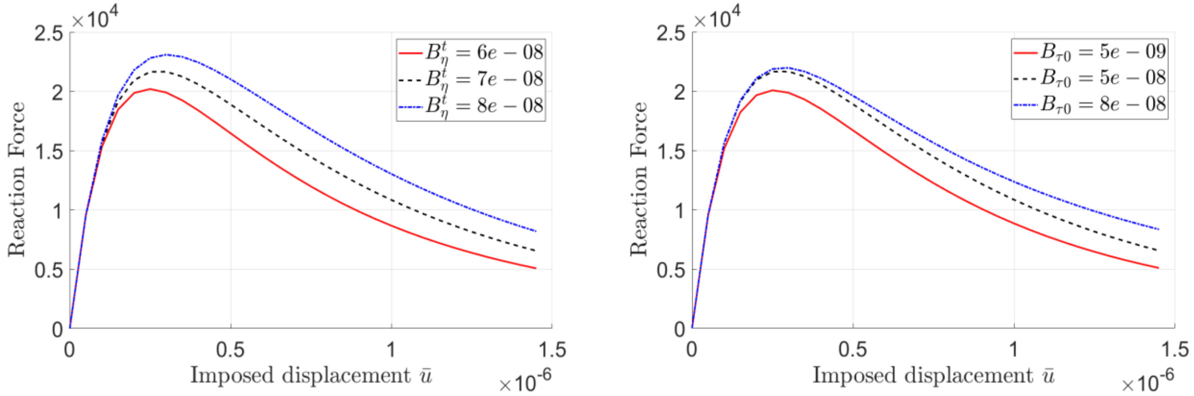
Polar plots of the damage variable D_τ for homogeneous compression and shearing tests and for increasing $\bar{u}$, when $B_\tau = B_{\tau 0} = const$ is considered. Black arrows indicate directions of increasing $\bar{u}$.

Results

RESULTS: HOMOGENEOUS CASE

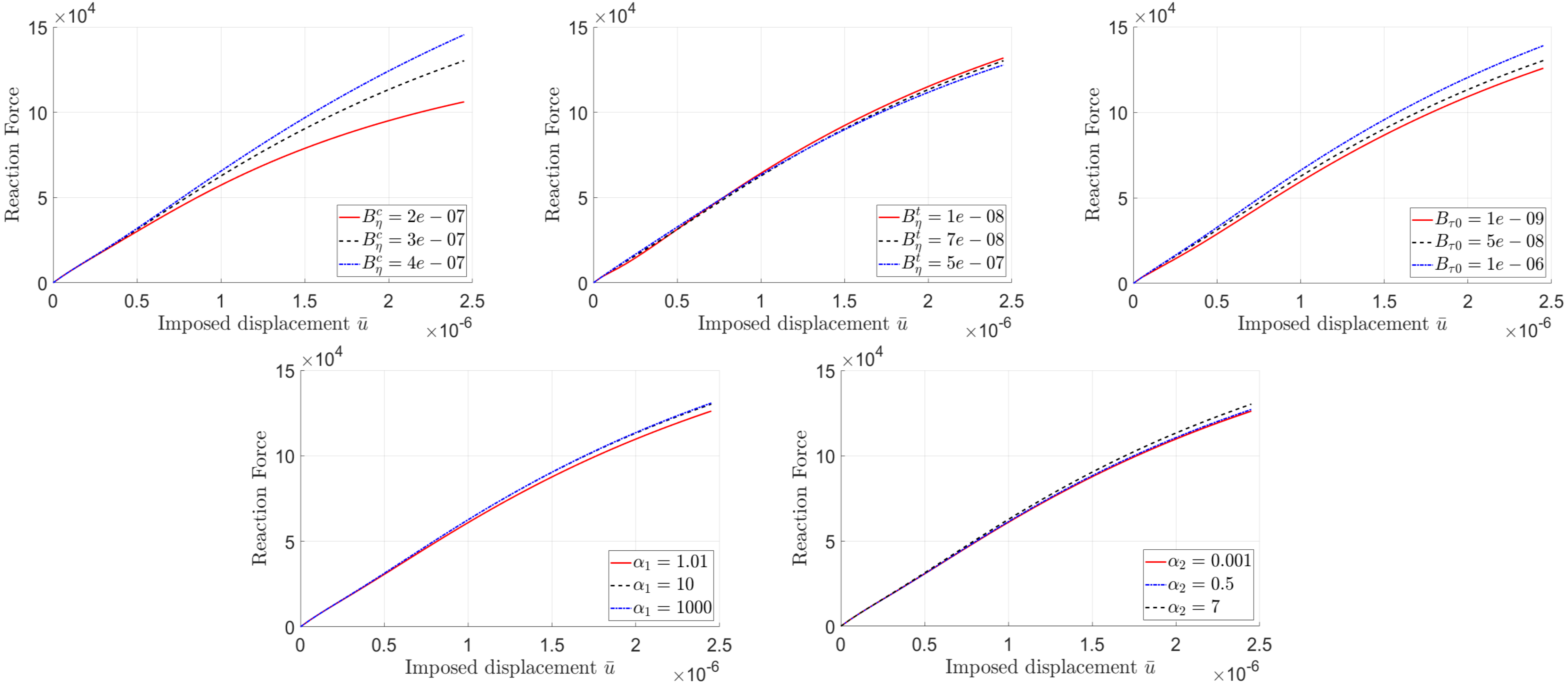


Compression test



Extension test

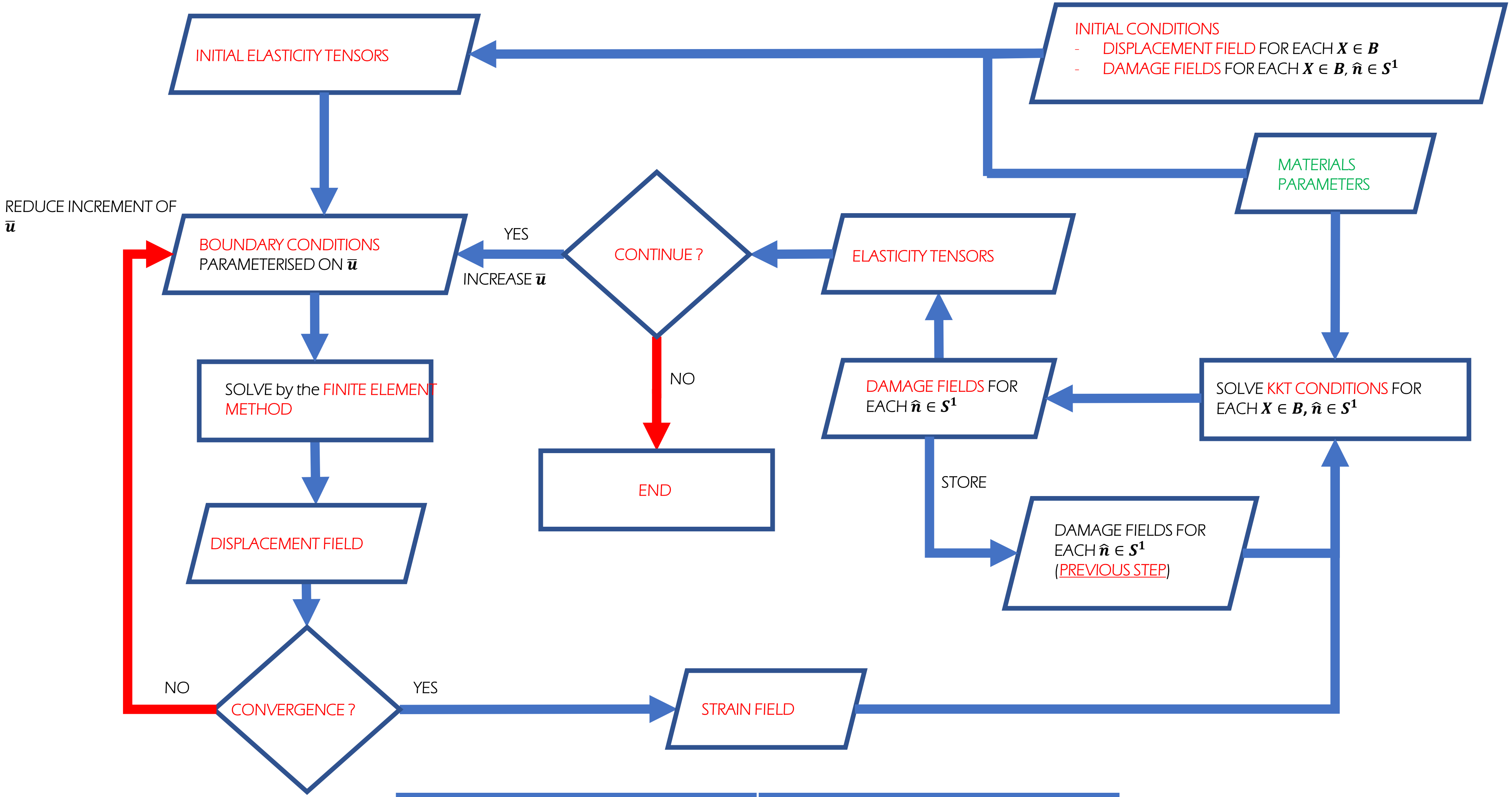
RESULTS: HOMOGENEOUS CASE



Shearing test

Results

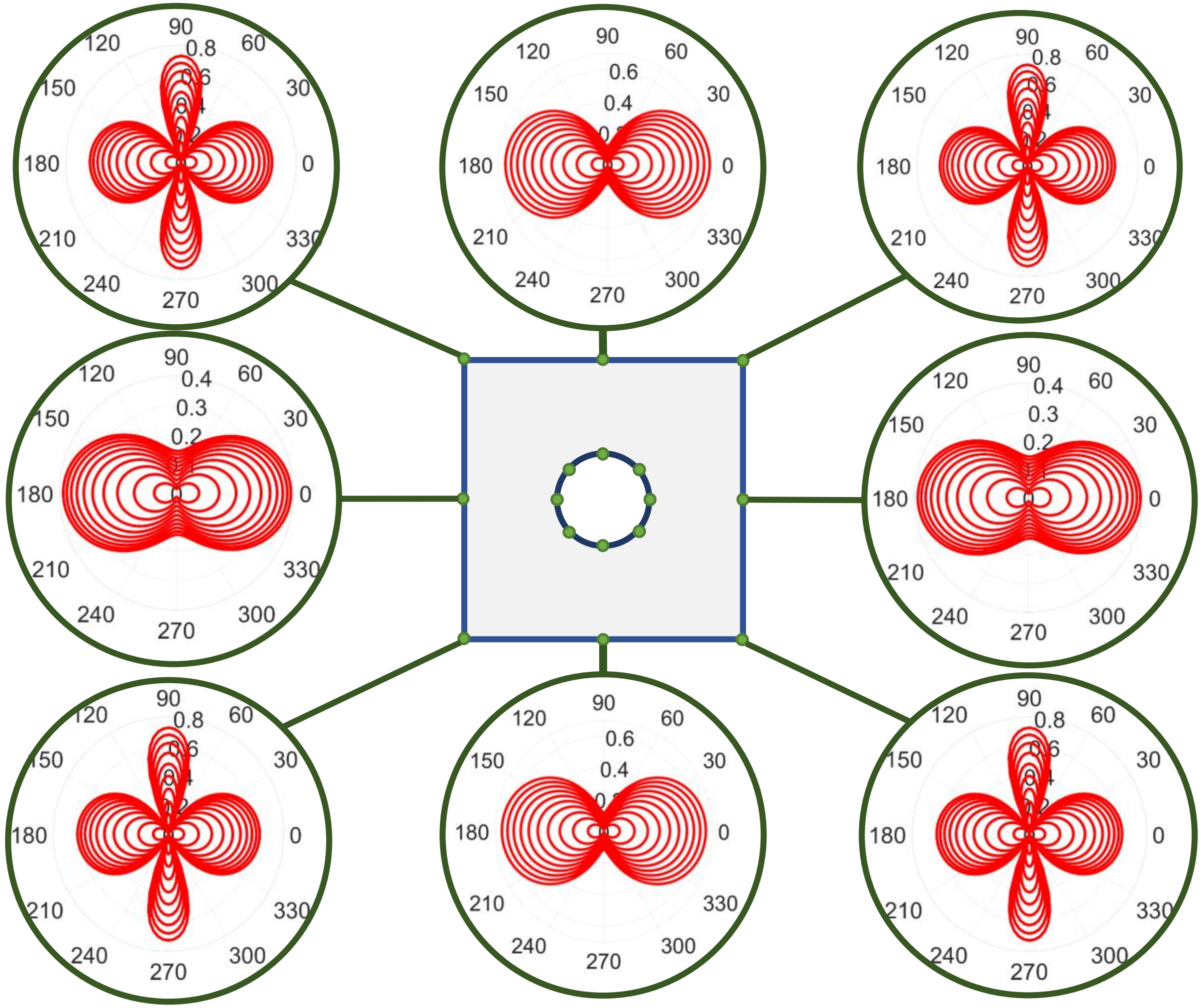
RESULTS: NUMERICAL ALGORITHM



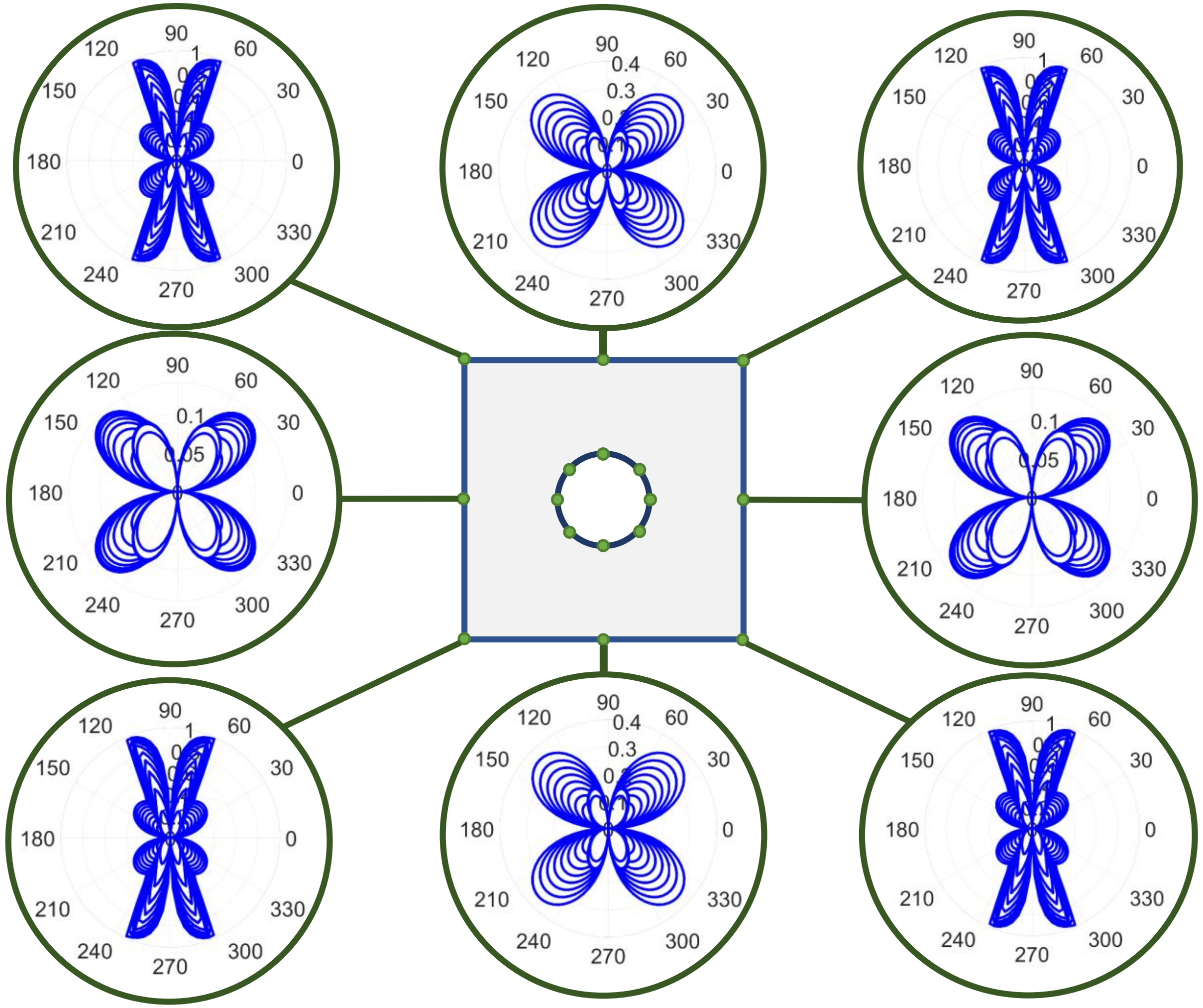
Compression\Shear	Extension
50 iterations	16 iterations

Results

RESULTS: DAMAGE IN COMPRESSION (EXTERNAL BORDER)

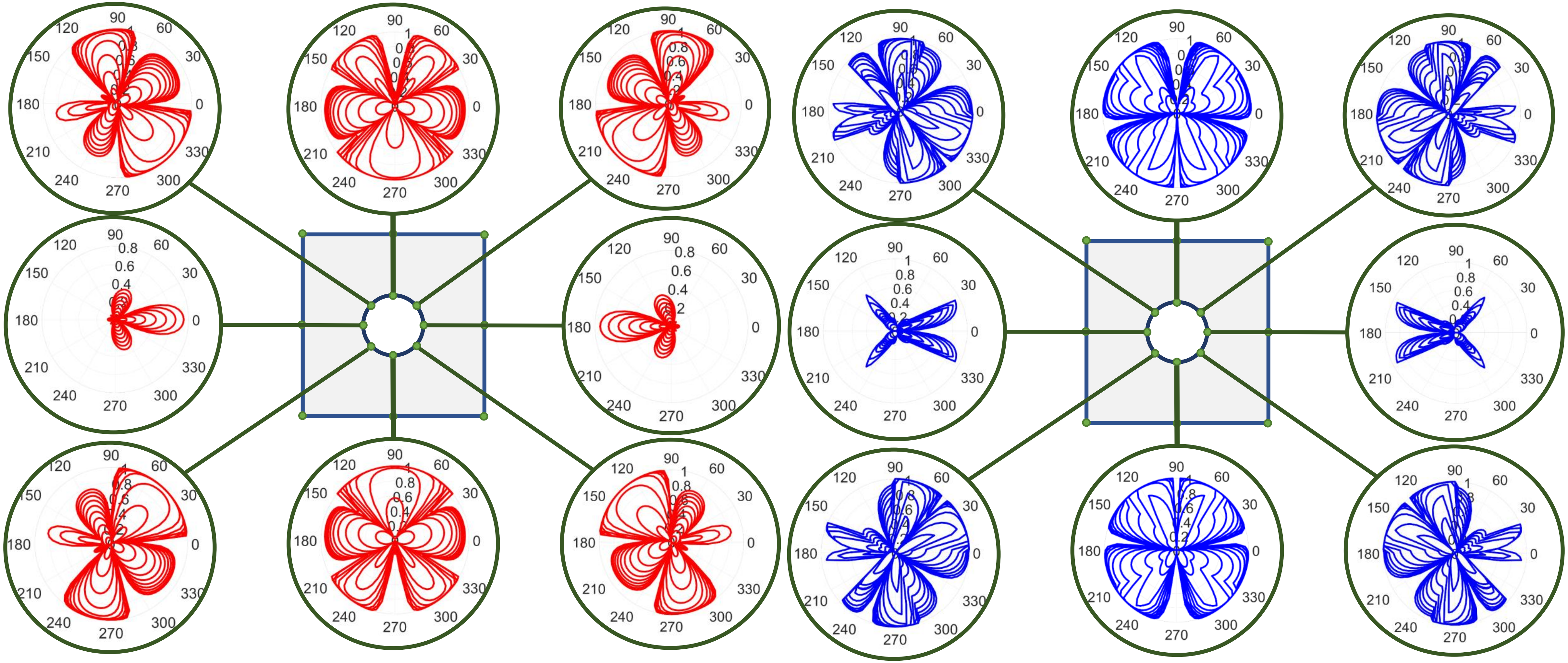


Normal damage



Tangent damage

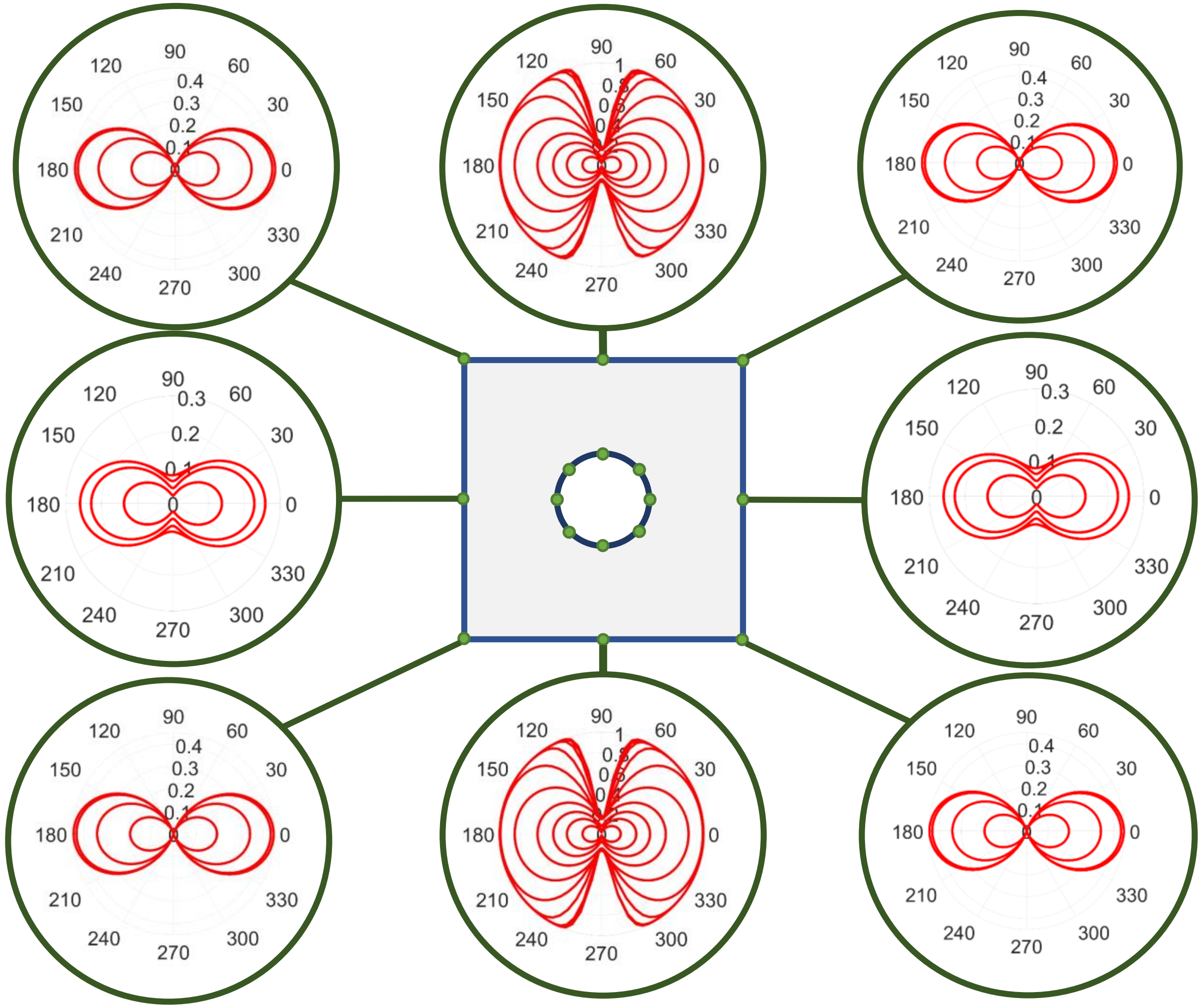
RESULTS: DAMAGE IN COMPRESSION (INTERNAL BORDER)



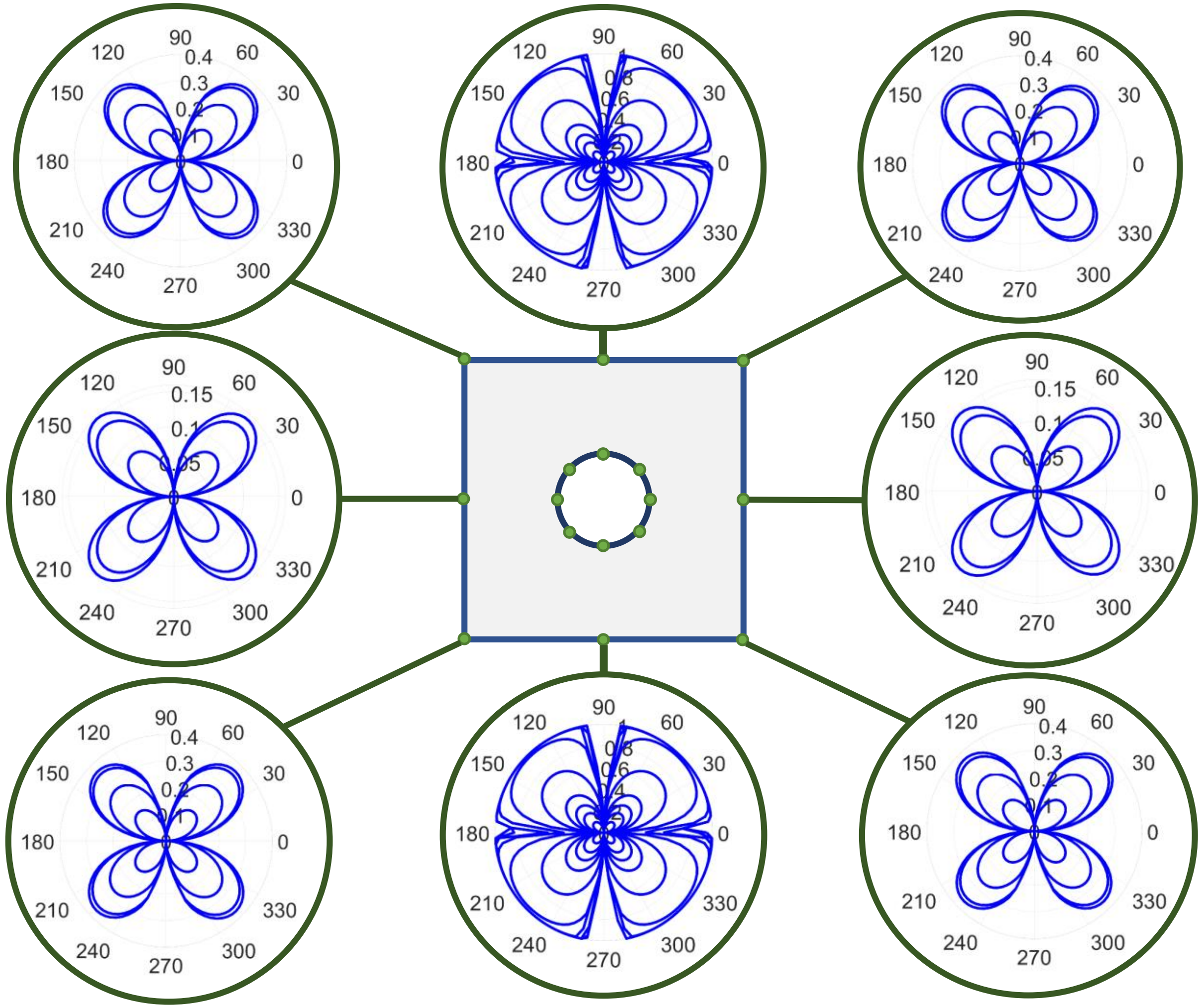
Normal damage

Tangent damage

RESULTS: DAMAGE IN EXTENSION (EXTERNAL BORDER)

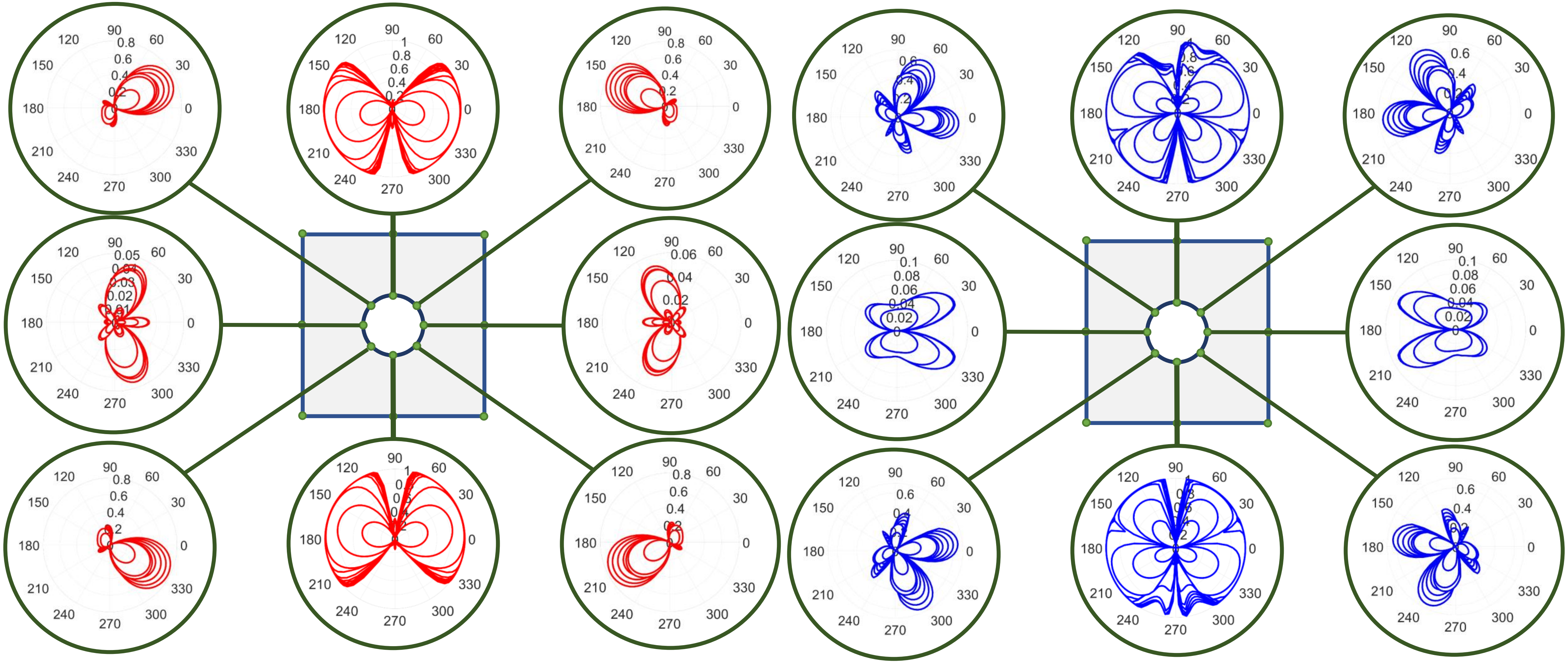


Normal damage



Tangent damage

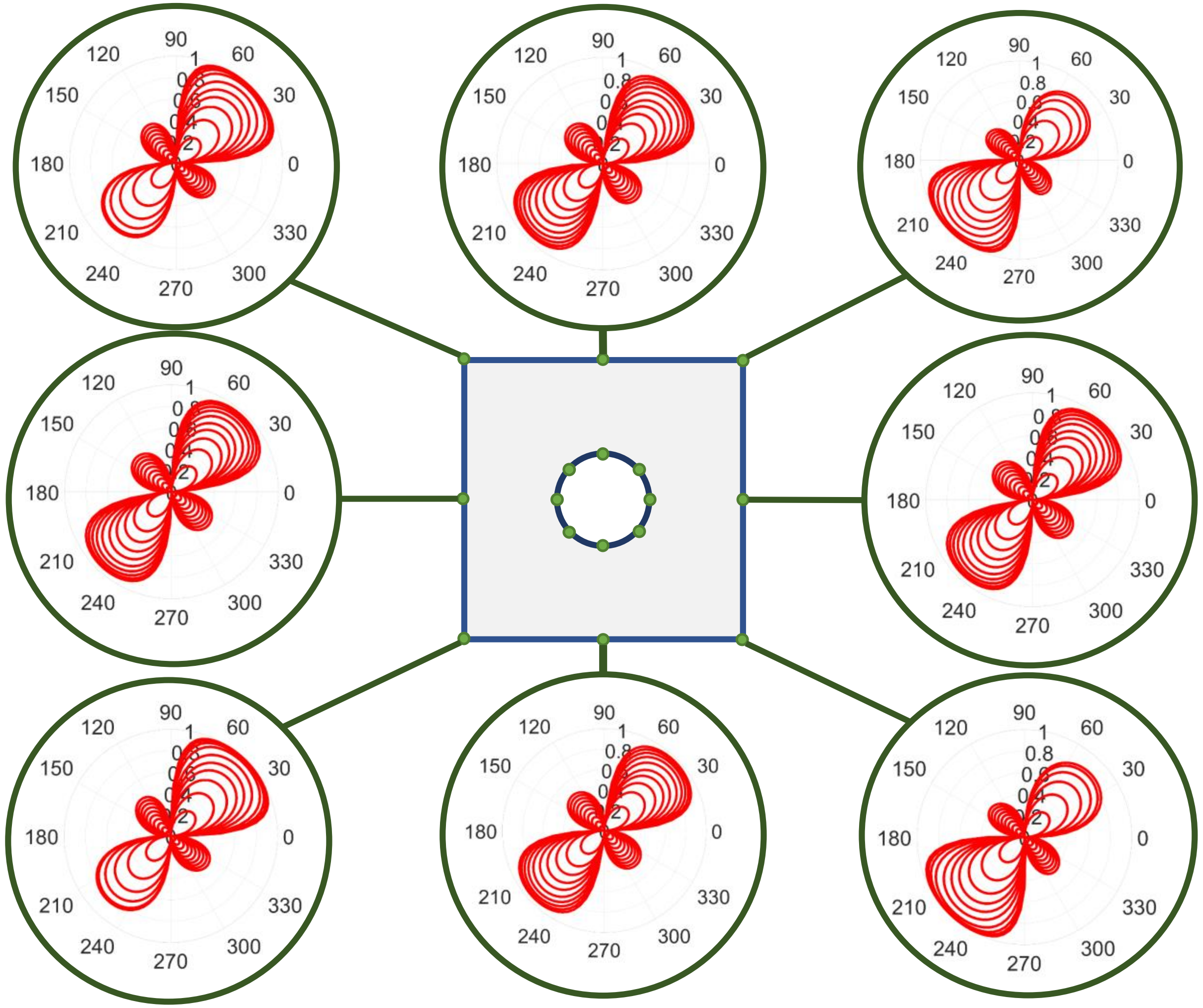
RESULTS: DAMAGE IN EXTENSION (INTERNAL BORDER)



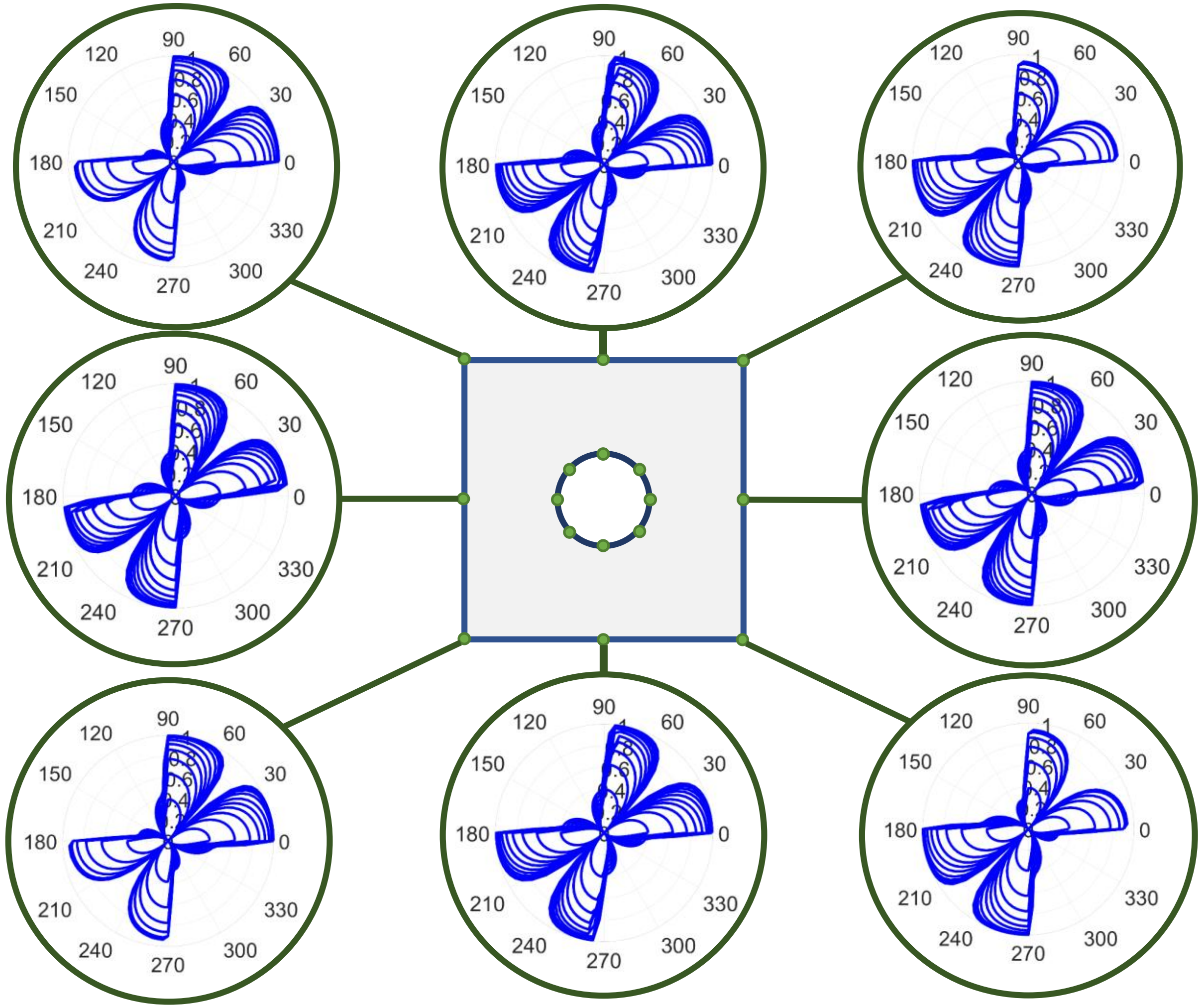
Normal damage

Tangent damage

RESULTS: DAMAGE IN SHEAR (EXTERNAL BORDER)



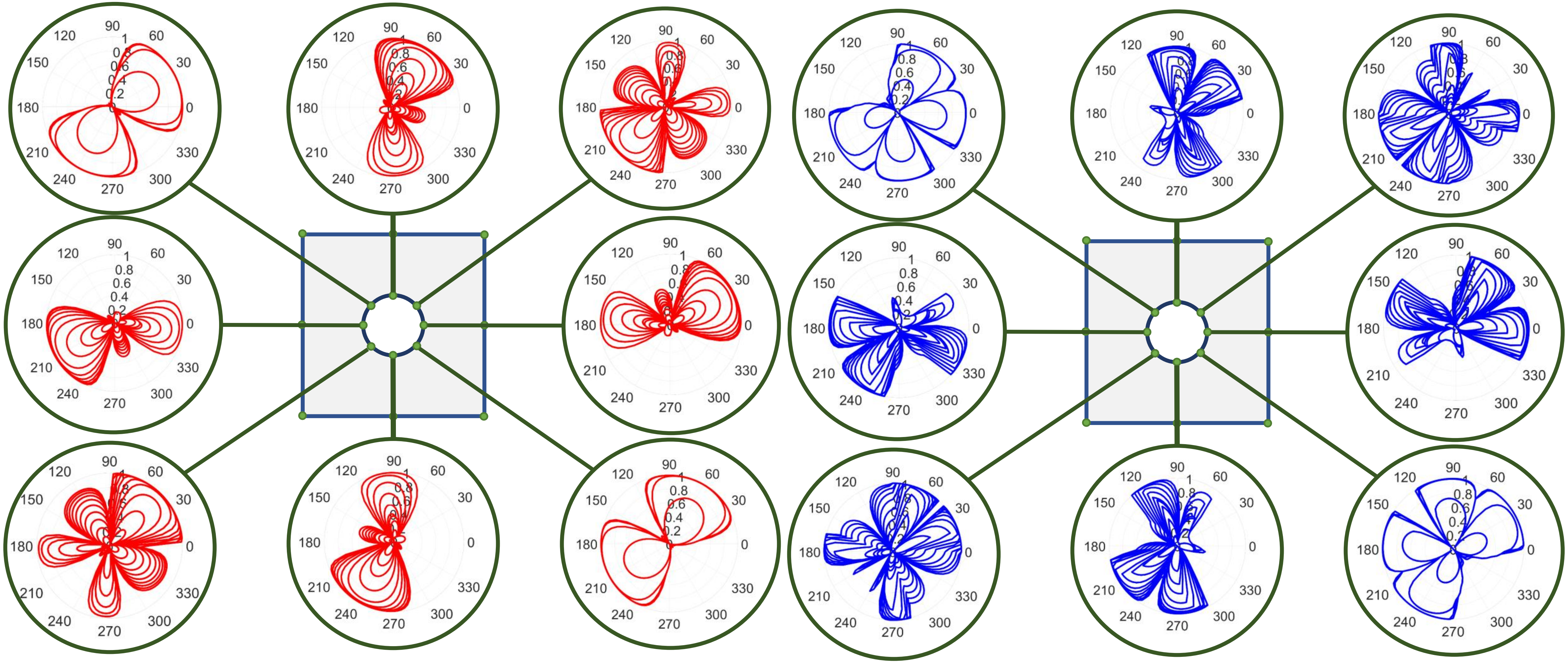
Normal damage



Tangent damage

Results

RESULTS: DAMAGE IN SHEAR (INTERNAL BORDER)



Normal damage

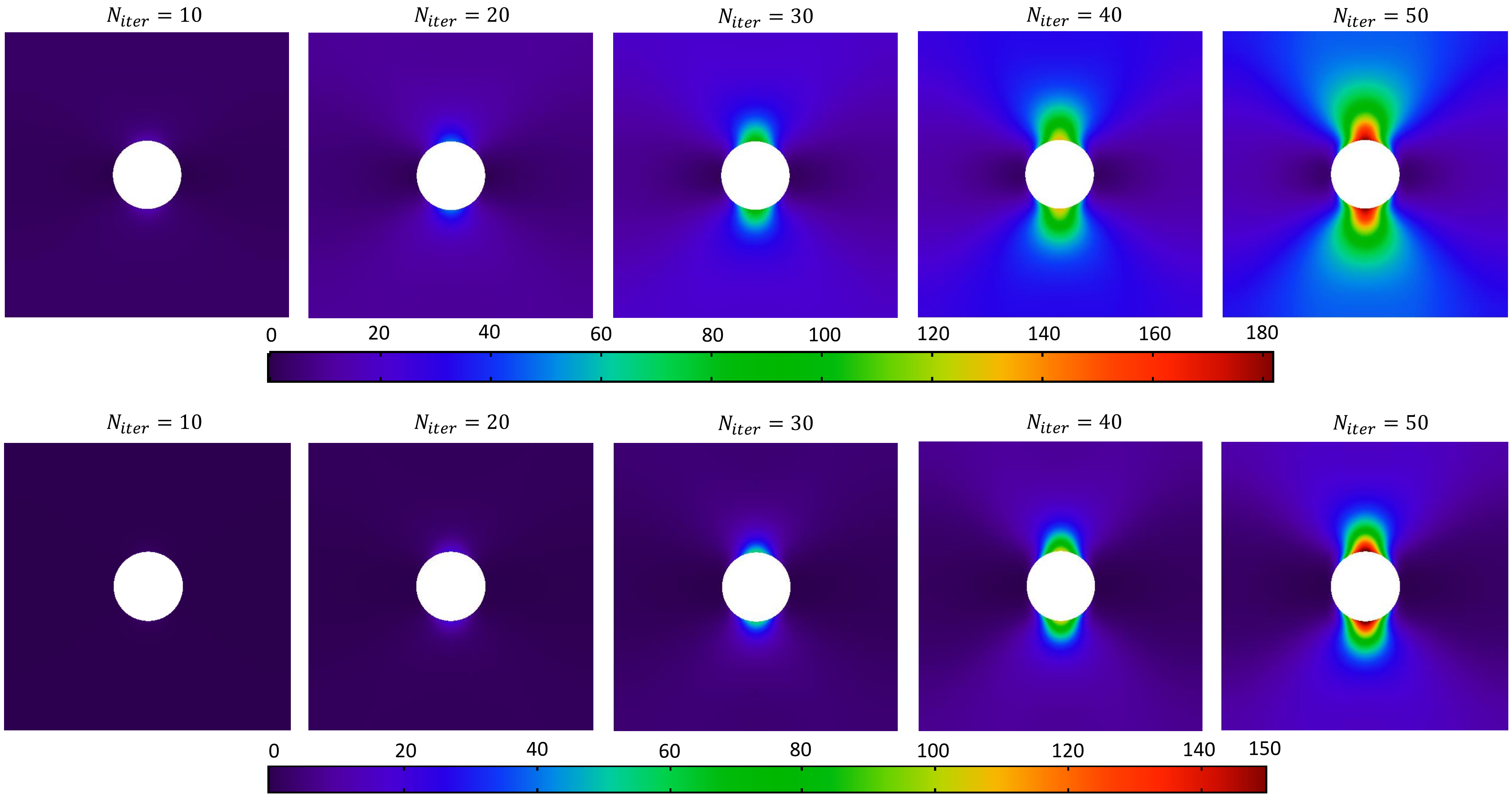
Tangent damage

Results

RESULTS: ELASTIC ENERGY IN COMPRESSION

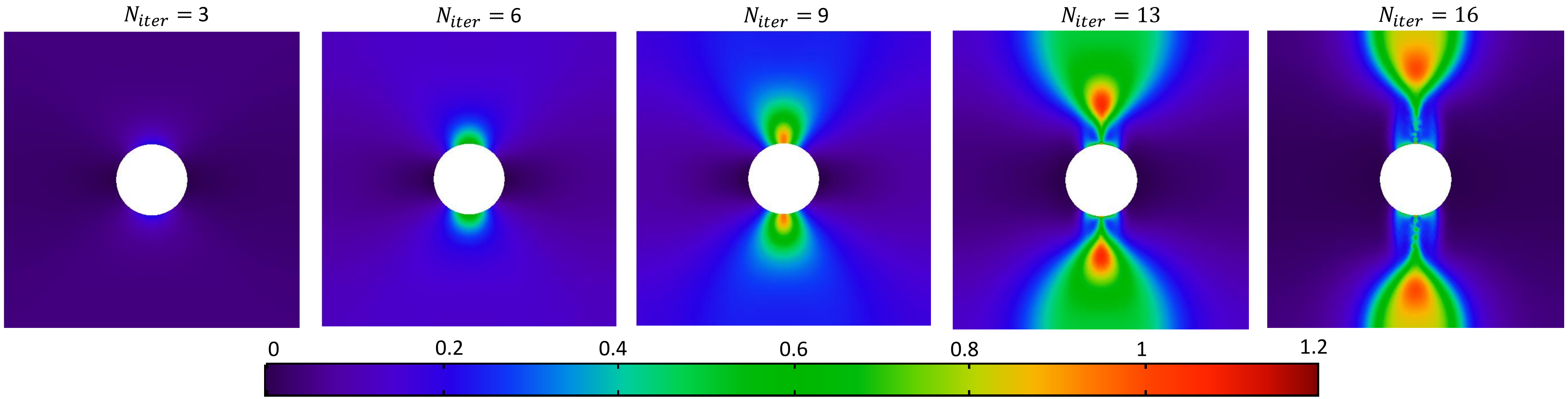
Elastic energy

Dissipation energy

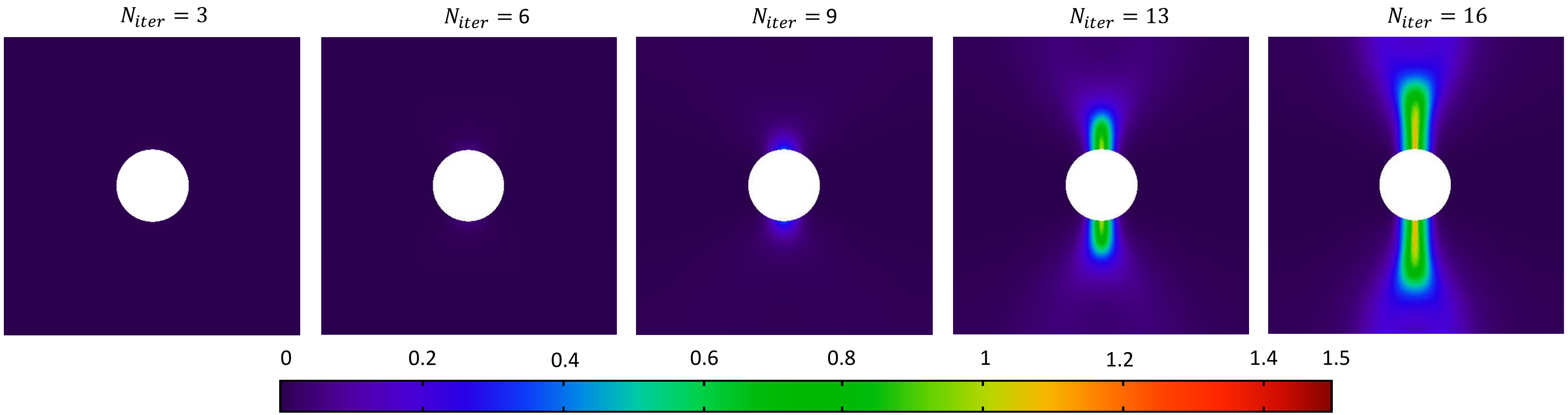


RESULTS: ELASTIC ENERGY IN EXTENSION

Elastic energy



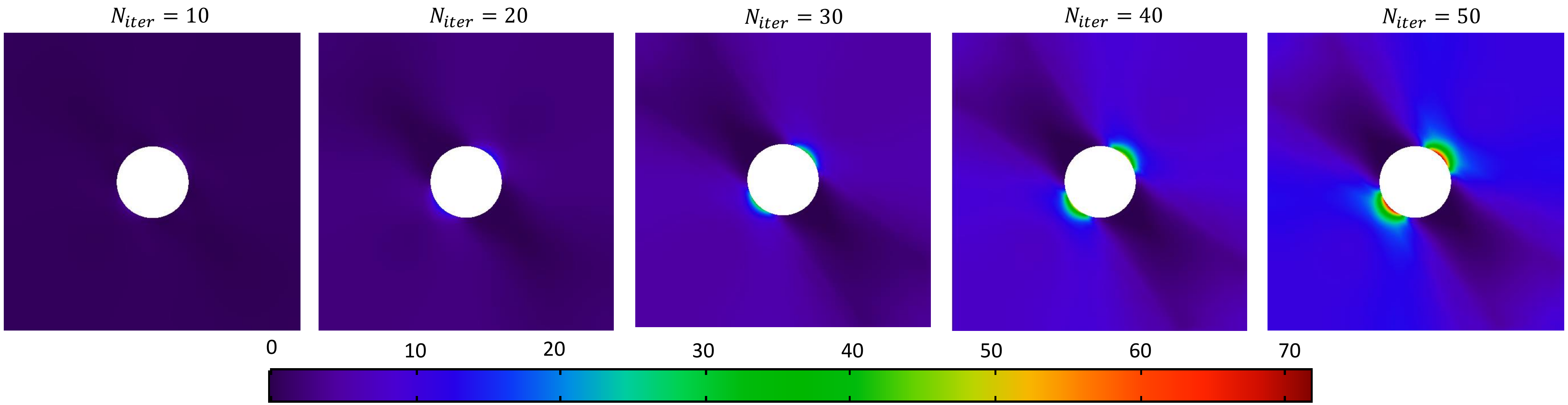
Dissipation energy



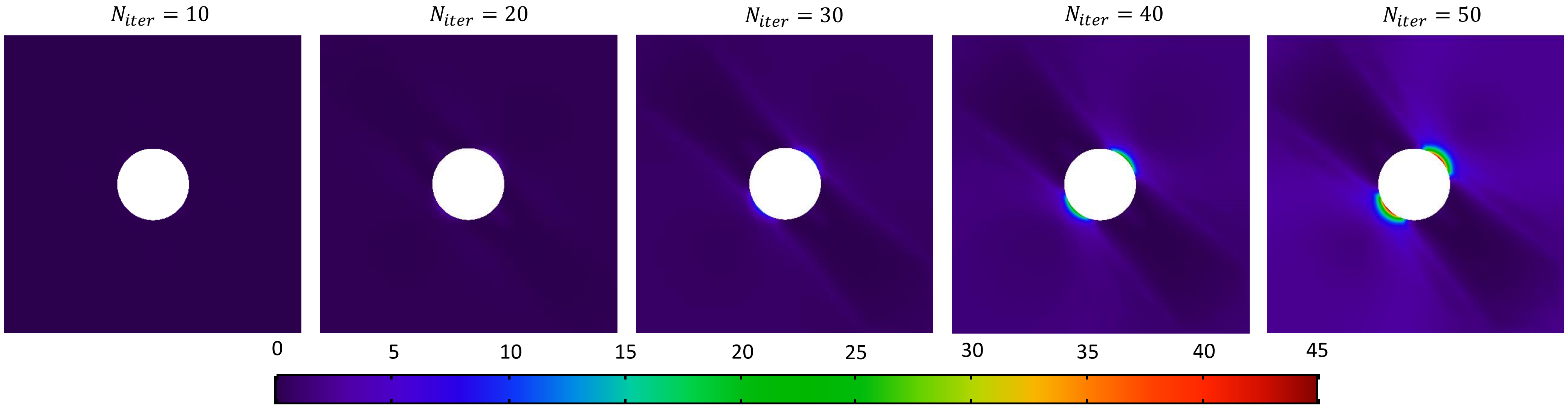
Results

RESULTS: ELASTIC ENERGY IN SHEAR

Elastic energy



Dissipation energy



CONCLUSION

- A simple method for the identification of isotropic and anisotropic elastic coefficients ($\mathbb{C}$, $\mathbb{M}$ and $\mathbb{D}$) for standard and strain grain gradient elastic coefficients all in terms of distance *L* and normal k_η and tangential k_τ elastic stiffness of grain pairs as well as of their distributions with respect to the orientation

- Grain-pairs oriented in different directions experience different loading histories, and therefore, different damage evolution for the normal D_η and the tangent D_τ components leading to damage-induced anisotropic response of the continua.

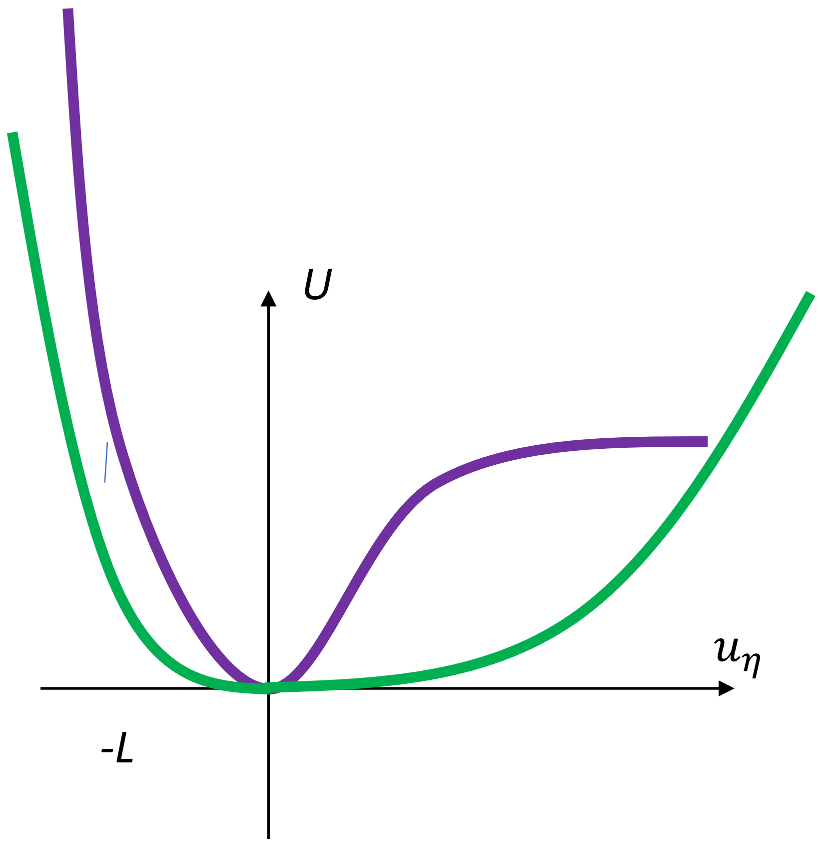
- Besides, erstwhile isotropic and non-chiral materials transform into anisotropic materials with chirality.

- The same for grain-pairs positions: for non-homogeneous deformations, every material point of a continuum evolves in a different way leading to damage-induced non-homogeneous continua.

- Tension-compression asymmetric behavior of grain-pair are easily modelled both in elastic and in damage contexts.

FUTURE WORK

- Generalize the simple **quadratic form** of elastic strain energy at grain level: for example with the use of **Leonard-Jones-type** potential in order to model elastic hardening.



- Generalize elastic strain energy at grain level with the gradient of relative displacement (e.g., with pantographic interactions). Other grain-pairs-stiffnesses, that necessarily degrade differently, will be considered. This should induce boundary layers even larger than *L*.

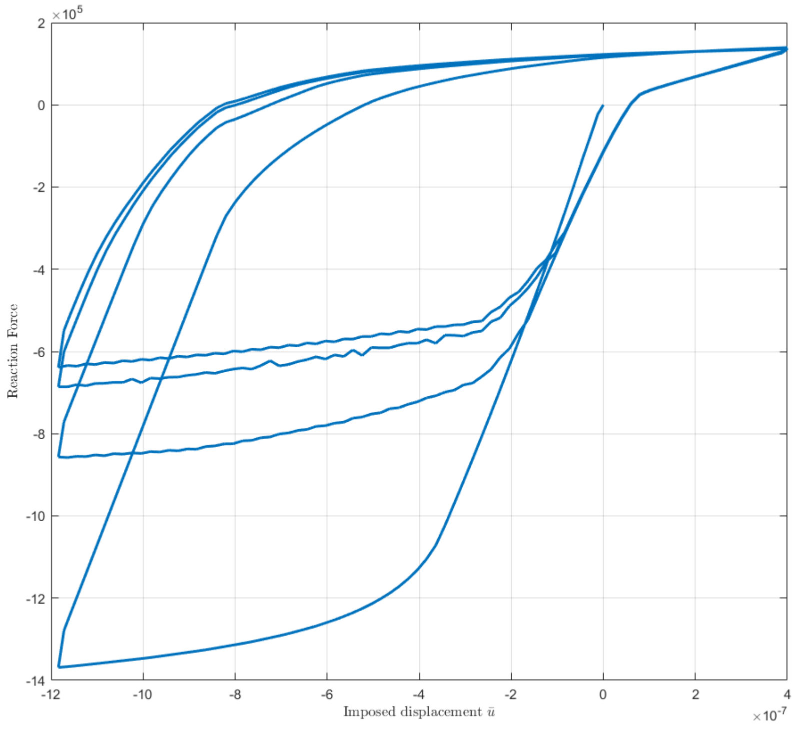
FUTURE WORK

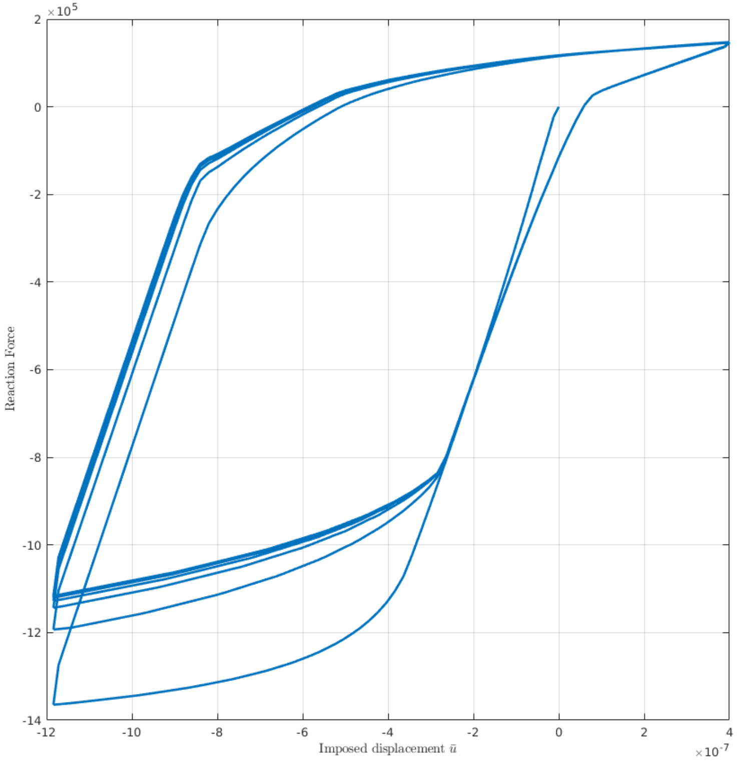
- Add kinematical descriptors at microscale and induce Cosserat and/or micromorphic continuum target theories.
- Generalization of the results to dynamics and to 3D.
- Experimental identification of those newly introduces dissipative coefficients

Conclusions and future works

FUTURE WORK

- Extend the proposed model to capture plasticity at the grain level. Plasticity induce a change in the stress-free configuration.





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THANK YOU FOR YOUR ATTENTION!