High-frequency homogenisation in 1D periodic media with imperfect interfaces of the spring-mass type

Raphael Assier, Marie Touboul^{*}, Bruno Lombard, and Cédric Bellis¹

¹Laboratoire de Mécanique et d'Acoustique (LMA) – Ecole Centrale de Marseille, Université de la Méditerranée - Aix-Marseille II, Université de Provence - Aix-Marseille I, CNRS : UPR7051 – France

Abstract

In this work, the concept of high-frequency homogenisation is extended to the case of one-dimensional periodic media that have an imperfect interface at the edges of the periodic cell. Indeed, when considering the propagation of elastic waves, displacement and stress discontinuities of the spring-mass types are allowed across the borders of the periodic cell. The high-frequency homogenisation of such media is carried out about the edges of the Brillouin zone that do not correspond to angular frequencies that are small, contrary to the classical low-frequency homogenization framework. At these edges, the dispersion diagram displays band-gaps, i.e. regions in the angular frequency space where waves do not propagate. A two-scale asymptotic expansion method is applied in order to approximate how the dispersion relation will behave around these zones. Asymptotic expansions are thus provided for the higher branches of the dispersion diagram and the resulting wavefield. The limiting case of two branches of the dispersion diagram that intersect with a non-zero slope (Dirac point) is also studied. The intermediate case of narrow bandgaps is also considered in order to obtain a uniform approximation that remains valid in the Dirac point limit. The examples of a monolayerd and a bilayered material are treated by this homogenised approach and a Bloch-Floquet approach in order to illustrate the validity of the method presented.

^{*}Speaker